TEST DGM #2

Solve the following tasks and send the solution to me by e-mail (`dolejsi@karlin.mff.cuni.cz`). It is sufficient to write the solution by hand on the paper, scan or make a snap by hand-phone. The references correspond to those in Lecture Notes
`http://msekce.karlin.mff.cuni.cz/~dolejsi/Vyuka/LectureNotes_DGM.pdf`

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1. In the definition of the numerical flux (2.15), we have used an extrapolation (2.17) for the boundary state, i.e.,

$$u_\Gamma^{(R)} := u_\Gamma^{(L)}, \qquad \Gamma \in \mathcal{F}_h^B. \tag{1}$$

   Another (natural) possibility is to put

$$u_\Gamma^{(R)} := u_D, \qquad \Gamma \in \mathcal{F}_h^B \cap \partial\Omega_D. \tag{2}$$

   Is Lemma 2.6 valid also for choice (2)? Is there any change in the proof?

2. Why Lemma 2.6 does not hold for the case $\Gamma_N \neq \emptyset$? Which step in the proof would be incorect?

3. Let us consider problem (2.1) with $\varepsilon = 0$, i.e., the pure hyperbolic problem without the diffusion. Does Theorem 2.14 give an appriori error estimate of this special case?

4. Derive the estimate of the interelement discontinuities of the approximate solution $u_h \in S_{hp}$, i.e. the estimate of the term

$$\int_0^T \sum_{\Gamma \in \mathcal{F}_h^I} \int_\Gamma [u_h(t)]^2 \,\mathrm{d}S \,\mathrm{d}t, \qquad t \in (0,T). \tag{3}$$

   Hint: direct use of estimate (2.83) in Theorem 2.14.

5. Comparing numerical results in Figures 2.1–2.2, we find that DG approximate solutions do not satisfy the homogeneous Dirichlet boundary condition $u = 0$ on $\partial\Omega$. Why? Add a few comments, if it is wrong in principle and if it is in contradiction with the main Theorem 2.14.

If you have any question, do not hesitate to contact me!