TEST DGM #3

Solve the following tasks and send the solution to me by e-mail (`dolejsi@karlin.mff.cuni.cz`). It is sufficient to write the solution by hand on the paper, scan or make a snap by hand-phone. The references correspond to those in Lecture Notes
`http://msekce.karlin.mff.cuni.cz/~dolejsi/Vyuka/LectureNotes_DGM.pdf`

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1. Let us consider the nonstationary heat equation

$$\frac{\partial u}{\partial t} - \varepsilon \Delta u = f. \tag{1}$$

   We discretize it by

   (i) BDF-DGM with the n-step backward difference formula

   (ii) STDGM with polynomial approximation degree q with respect to the time

   Both discretization leads to linear algebraic systems on each time level.

   - How depends the size of these systems (=number of unknowns and equations) on n and q?
   - Which of this technique is more efficient (from the point of view of the computational time necessary to perform one time step)?

2. Both techniques (BDF-DGM and STDGM) were derived for the constant time step τ. Which of these techniques can be easily adopted to variable time steps?

3. Both techniques (BDF-DGM and STDGM) were derived for the same mesh on each time level. Which of these techniques can be easily adopted to variable meshes? Why?

4. *This task requires some knowledge about the stability of numerical methods for ODEs. If you are not familiar with this subject, you need not to solve this task.* It is known fact that the n-step BDF method is unconditionally stable for $n = 1, 2$. However, in the semi-implicit discretization (3.8) for the convection-diffusion equation (3.1), we employed the extrapolation (3.9).

   - Can we expect that the unconditional stability will be preserved (or decreased)?
   - How does this property depend on the size of ε in (3.1)?

If you have any question, do not hesitate to contact me!