Explicit Euler method

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Local error

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Approximation of y''

$y''(\cdot) \approx y''_{k-1} := \frac{y'_k - y'_{k-1}}{x_k - x_{k-1}} = \frac{f(x_k, y_k) - f(x_{k-1}, y_{k-1})}{x_k - x_{k-1}}, \quad k = 1, 2, \ldots$

- the goal: $L_k \leq \mathrm{TOL}, \quad \mathrm{TOL} > 0$ is given,
- the idea: set h_k such that $L_k \approx \mathrm{TOL}$.

Optimal size of the time step

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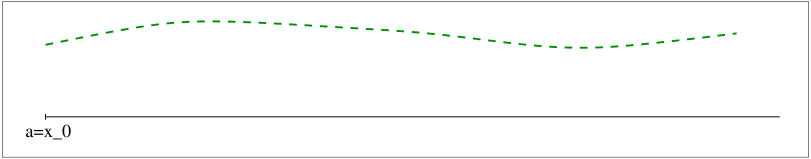
adaptive choice of time step "guarantees" the stability of method



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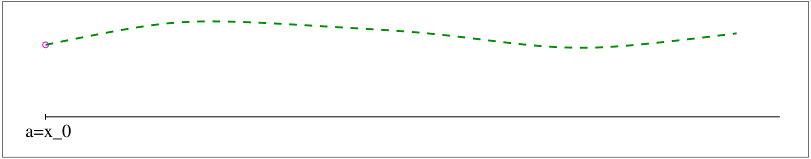

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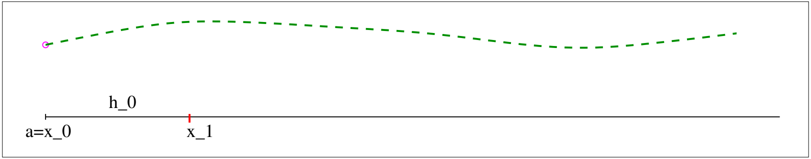

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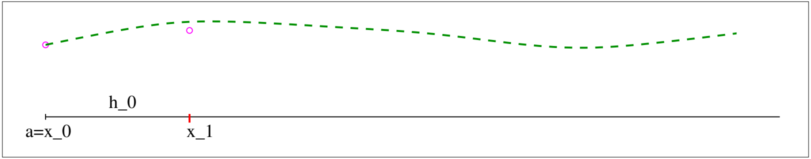
Explicit Euler methods, simple adaptive choice of the time step ($L_k \leq \mathrm{TOL}$)



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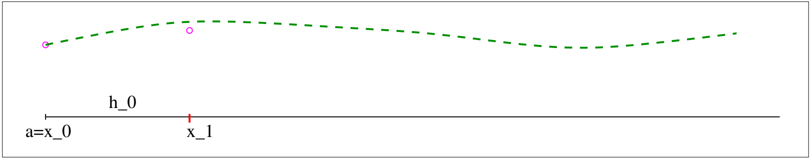

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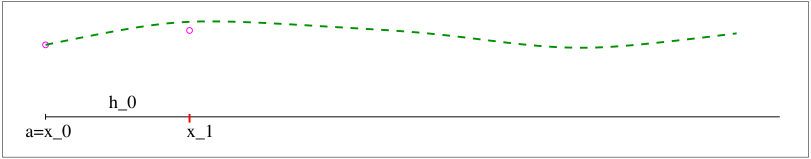

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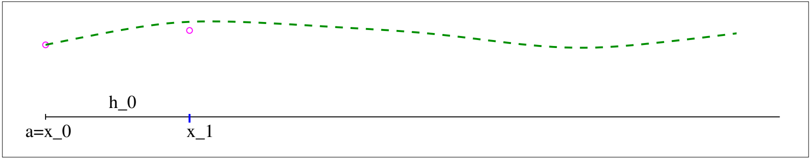

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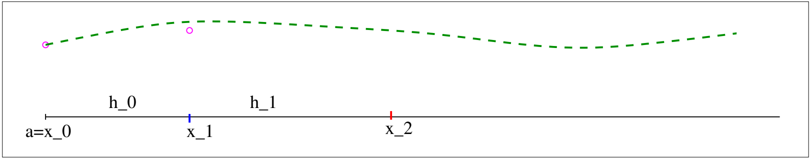

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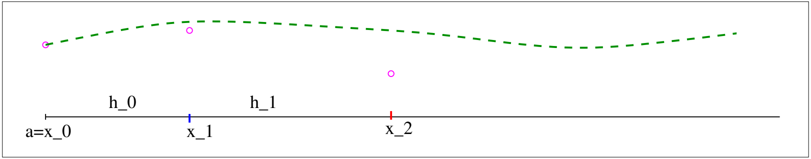

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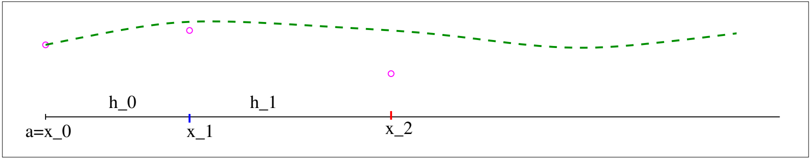

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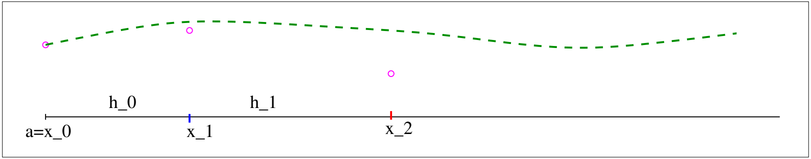

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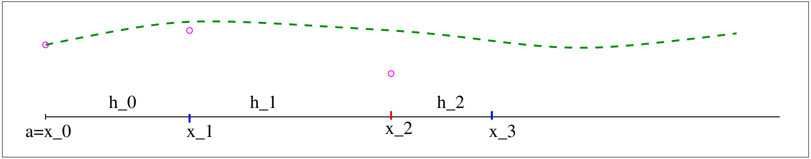

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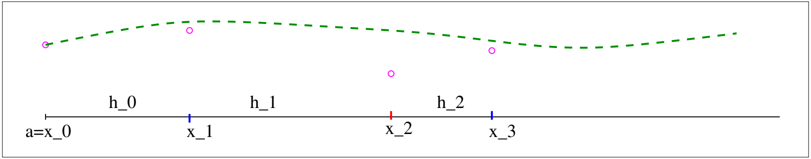

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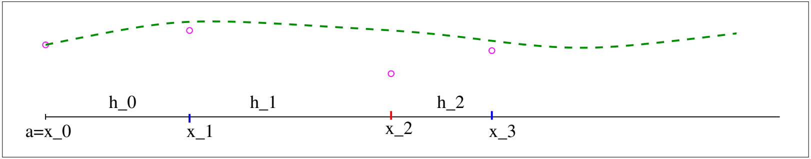

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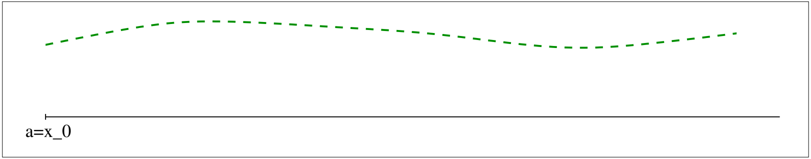

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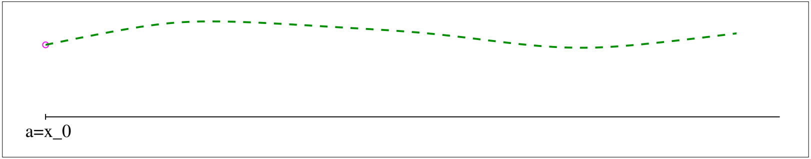

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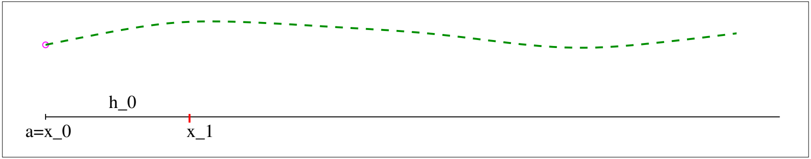

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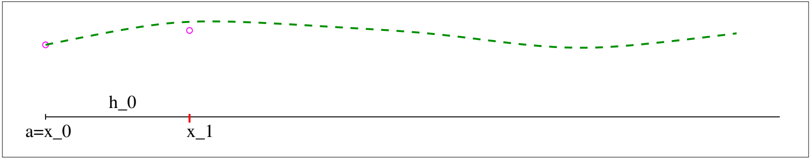

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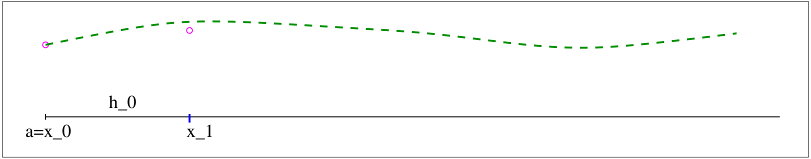

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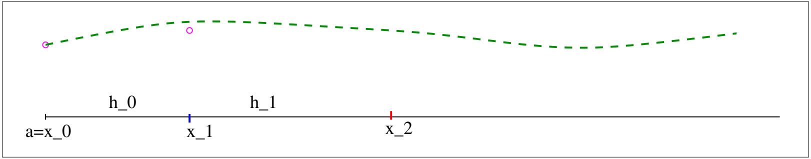

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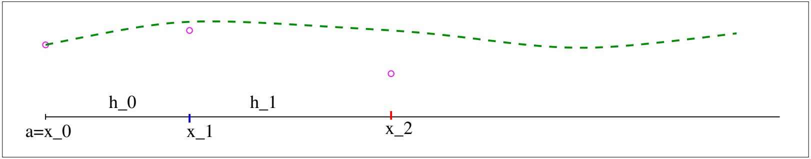

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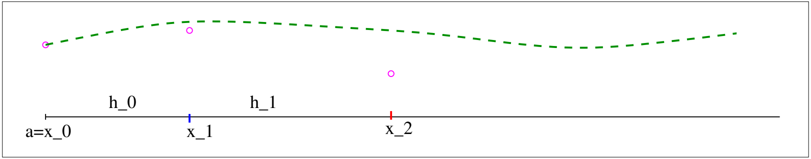

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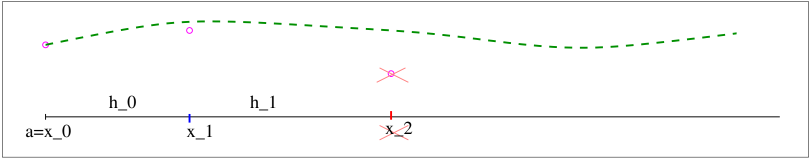

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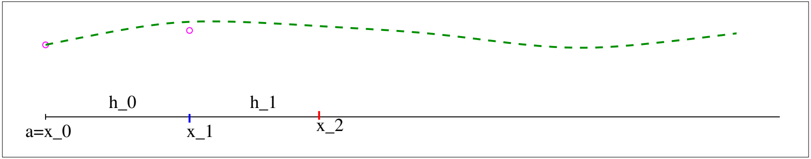

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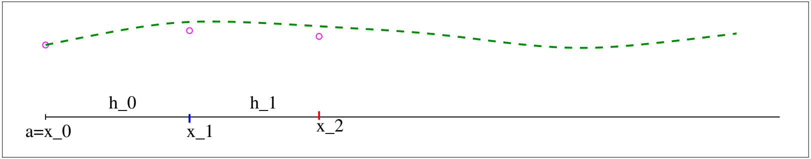

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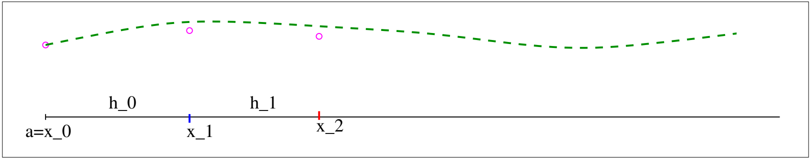

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