Write a simple code which computes numerically the integral $\int_a^b f(x)\,\mathrm{d}x$ using

- composite **midpoint** rule $M_h(f)$,
- composite **trapezoid** rule $T_h(f)$,
- composite **Simpson** rule $S_h(f)$.

Show by numerical examples the following items:

(i) the order of the corresponding quadrature is p, i.e., $Q_h(f)$ is exact for polynomials of degree p,

(ii) the order of the corresponding composite quadrature is p, i.e., $I(f) - Q_h(f) = O(h^{p+1})$,

(iii) test and explain, why $I(f) - Q_h(f) = O(h^{p+1})$ is not true for $\int_0^1 \sqrt{x}\,\mathrm{d}x$?

(iv) the error estimate of the midpoint formula by the relation

$$E_h \approx \tfrac{1}{3}(M_h(f) - T_h(f)) \tag{0.1}$$

for smooth and non-smooth functions.

(v) find an example where the estimate (0.1) fails (for the given h only)

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code is available on `http://msekce.karlin.mff.cuni.cz/~dolejsi/Vyuka/NS_source/Quad/NC_test.tgz`

- download the code on unpack it by the command `tar xfz NC_test.tgz`, directory `NC_test/` will appear
- translate the code `NC.f90` by `make` (the corresponding `Makefile` is attached)
- study the code and modify it in order to solve the tasks given above

Write a code for the **local** and **global** error estimate using the **half-step size method** for the midpoint, trapezoid and Simpson rules:

$$\mathrm{EST}_{R,h/2} = \frac{|Q_{R,h} - Q_{R,h/2}|}{2^{p+1} - 1}$$

where

- $Q_{R,h}$ – result by the quadrature Q $(= M, T, S)$ with step h over interval R
  - $R = (a, b)$ – global variant
  - $R = (x_i, x_{x+1})$ – local variant
- p – order of the method

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1. Modify the code `NC.f90` from the previous tutorial,
2. test the method for the regular problem $(\int_0^1 x^4 \,\mathrm{d}x)$ and a singular problem $(\int_0^1 \sqrt{x}\,\mathrm{d}x)$,
3. compare the global estimator (EST) with the real error (ERR); effectivity index $i_{\mathrm{eff}} = \frac{\mathrm{EST}}{\mathrm{ERR}}$,
4. plot the distribution of the local estimators and it comparison with the local real error.

Possible solution is the code `NC2.f90` in the previous archive

Output files:	midpoint	trapezoid	Simpson	**gnuplot** command	meaning
$h \to h/2$	fort.12	fort.22	fort.32	**p 'fort.**' u 1:2 w l**	local error estimate distribution
$h/2 \to h/4$	fort.13	fort.23	fort.33	**p 'fort.**' u 1:3 w l**	local error distribution
$h/4 \to h/8$	fort.14	fort.24	fort.34		
$h/8 \to h/16$	fort.15	fort.25	fort.35		
global	fort.101	fort.102	fort.103	**p 'fort.***' u 2:3 w l**	error estimate vs. h
				p 'fort.*' u 2:4 w l**	error vs. h
				p 'fort.*' u 2:5 w l**	effectivity index vs. h