Write a simple code for the solution of problem: find $u : \Omega \to \mathbb{R}$ such that

$$
\begin{aligned}
-\Delta u(x) &= g(x), \quad x \in \Omega, & (0.1)\\
u(x) &= u_D(x), \quad x \in \partial\Omega_D, & (0.2)\\
\nabla u(x) \cdot \boldsymbol{n} &= g_N(x), \quad x \in \partial\Omega_N, & (0.3)
\end{aligned}
$$

where $\Omega$ is a domain in $\mathbb{R}^2$ with a boundary $\partial\Omega$ consisting of two disjoint parts $\partial\Omega_D$ and $\partial\Omega_N$, $g \in L^2(\partial\Omega_D)$, $g_N \in L^2(\partial\Omega)$ and $u_D$ is a trace of some $u^* \in H^1(\Omega)$.

- use $P_1$-conforming FE
- the arising linear system solve by a simple iterative method, e.g., Jacobi or Gauss-Seidl
- the stiffness matrix can be treated as dense

Use the code: `http://msekce.karlin.mff.cuni.cz/~dolejsi/Vyuka/NS_source/FEM/FEM-code.tgz` link

- `mesh.f90` – reading the mesh from the file `triang`
- `matrix.f90` – create the stiffness matrix, solution of $\mathbb{A}x = b$
- `sol.f90` – setting of RHS and BC (input of data)
- `femP1.f90` – main code

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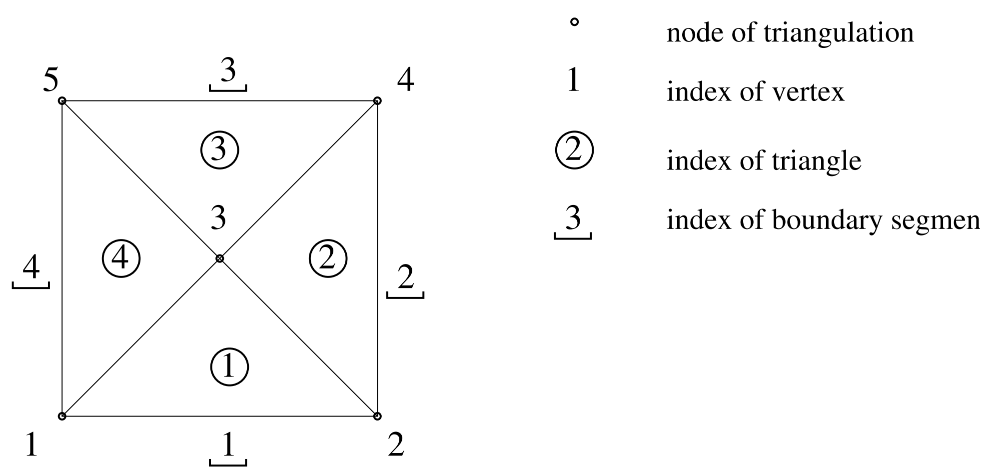
- type of boundary set in `subroutine Read_mesh`, file `mesh.f90`
- array `ip(:, 1)` – type of mesh vertices: `> 0` – interior, `= 0` – Neumann, `< 0` – Dirichlet,
- array `ip(:, 2)` – index of vertex after removing Dirichlet nodes

## Example of file `triang` for the unit square

```
5   4      4   4                   npoin nelem nbelm  nbc
0.0     0.0   0   0   0.0    0.0   0   0    periodicity
0.0     0.0                        xp(1)  yp(1)
1.0     0.0                        xp(2)  yp(2)
0.5     0.5                          .      .
1.0     1.0                          .      .
0.0     1.0                        xp(5)  yp(5)
1  2  3                            lnd(1,1)  lnd(1,2)  lnd(1,3)
2  4  3                              .      .
4  5  3                              .      .
5  1  3                            lnd(4,1)  lnd(4,2)  lnd(4,3)

1  2  1                            lbn(1,1)  lbn(1,2)  lbn(1,3)
2  4  2                              .      .
4  5  3                              .      .
5  1  4                            lbn(4,1)  lbn(4,2)  lbn(4,3)
```

the mesh of $\Omega = (0,1) \times (0,1)$



## Basic tasks

1. study the code line by line, is something is unclear ask the teacher
2. find what problem is solved by default
3. find how is solved the arising algebraic system
4. replace this subroutine by another one
5. modify the code such that the following boundary conditions are treated
   (a) homogeneous Dirichlet on the whole boundary
   (b) non-homogeneous Dirichlet on the whole boundary
   (c) combination of the Dirichlet and Neumann BC
   (d) Neumann BC on the whole boundary (troubles are expected)

## More advanced tasks

1. write a subroutine computing the error in the $L^2$-norm and $H^1$-seminorm (provided that the exact solution is known)
2. using computation on a sequence of meshes set the *experimental order of convergence*

**2D quadrature on triangle** (weights and the barycentric coordinates $n = 6$: order = 4)

$w_1 = 2.2338158967801100\text{E-}01$
$w_2 = 2.2338158967801100\text{E-}01$
$w_3 = 2.2338158967801100\text{E-}01$
$w_4 = 1.0995174365532200\text{E-}01$
$w_5 = 1.0995174365532200\text{E-}01$
$w_6 = 1.0995174365532200\text{E-}01$

$x_1(1:3) = ($ 1.0810301816807000E-01, 4.4594849091596500E-01, 4.4594849091596500E-01)
$x_2(1:3) = ($ 4.4594849091596500E-01, 4.4594849091596500E-01, 1.0810301816807000E-01)
$x_3(1:3) = ($ 4.4594849091596500E-01, 1.0810301816807000E-01, 4.4594849091596500E-01)
$x_4(1:3) = ($ 8.1684757298045896E-01, 9.1576213509771007E-02, 9.1576213509770035E-02)
$x_5(1:3) = ($ 9.1576213509771007E-02, 9.1576213509771007E-02, 8.1684757298045796E-01)
$x_6(1:3) = ($ 9.1576213509771007E-02, 8.1684757298045896E-01, 9.1576213509770035E-02)