

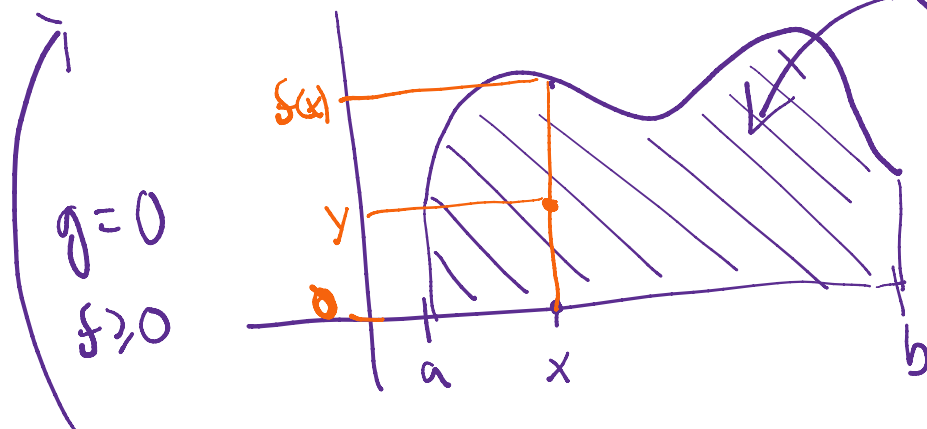
$$\int_0^{\frac{1}{2}} \frac{1}{x |\log x|^p} dx = \begin{cases} \text{konverguje} & p > 1 \\ \text{diverguje} & p \leq 1 \end{cases}$$

$$\int_2^{\infty} \frac{1}{x \log^p x} dx = \begin{cases} \text{konverguje} & p > 1 \\ \text{diverguje} & p \leq 1 \end{cases}$$

$$P(S, \underline{a}, b) = \int_a^b f$$

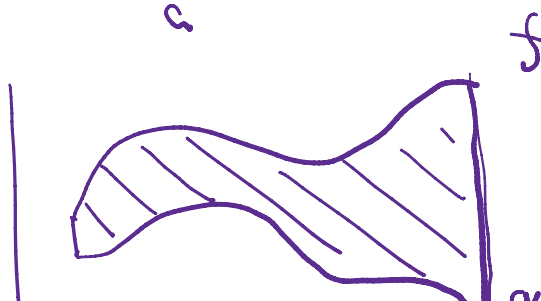
$$\boxed{f \geq 0}$$

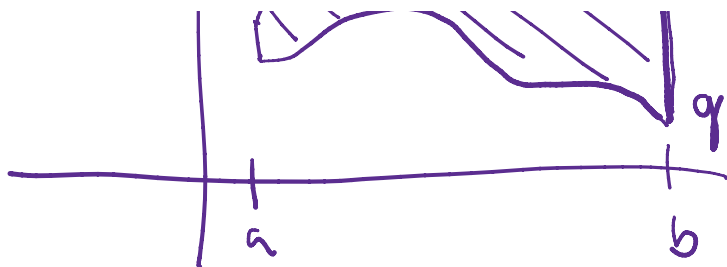
$$\{(x, y) : a \leq x \leq b, 0 \leq y \leq f(x)\}$$



$$P(S, \underline{g}, \underline{a}, b) = \int_a^b f - g$$

$$\underline{g \leq f}$$





$$f(x) = x \quad g(x) = x^4$$

plocha mezi grafy f a g

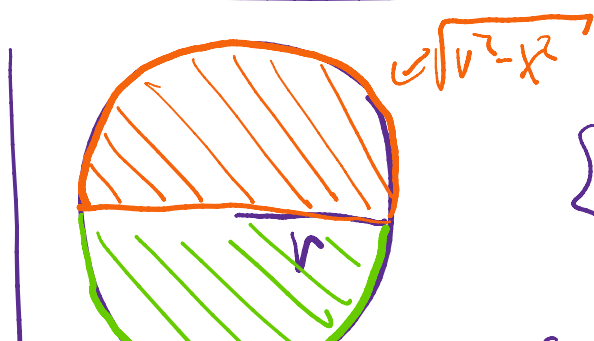


$$f(x) = g(x)$$

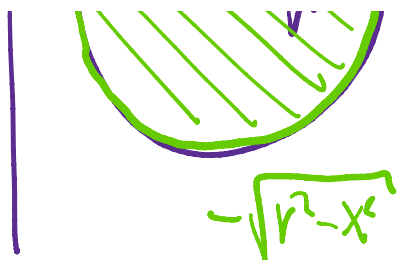
$$x = x^4 \quad x = 0, 1$$

$$\int_0^1 f - g = \int_0^1 x - x^4 dx = \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1 = \frac{1}{2} - \frac{1}{5} = \underline{\underline{\frac{3}{10}}}$$

• $M = \{(x, y) : \underline{a \leq x \leq b}, \underline{g(x) \leq y \leq f(x)}\}$



• $\{(x, y) : \underline{x^2 + y^2 \leq r^2}\}$



$$S(x) = \sqrt{r^2 - x^2} \quad x \in [-r, r]$$

$\nwarrow$ $x = r \sin t$

(3)

$$S(x) = \underline{\arcsin x} + \sqrt{1 - x^2} \quad x \in [-1, 1]$$

$\nwarrow$ $()^{\frac{1}{2}}$

$$\varphi(t) = (t, S(t))$$

$$L(\varphi) = \int_{-1}^1 \sqrt{\underline{1} + (S'(x))^2} dx$$

$$S'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-2x}{2\sqrt{1-x^2}} = \frac{1-x}{\sqrt{1-x^2}}$$

$$L(\varphi) = \int_{-1}^1 \sqrt{1 + \left| \frac{(1-x)^2}{1-x^2} \right|} dx = \int_{-1}^1 \sqrt{1 + \frac{1-x}{1+x}} dx$$

$$= \int_{-1}^1 \sqrt{\frac{\cancel{(1-x)}^2}{\cancel{(1-x)}(1+x)}} dx = \int_{-1}^1 \sqrt{\frac{2}{1+x}} dx$$

$\therefore \dots \therefore dk = dv$

$$= \sqrt{2} \int_{-1}^1 \frac{1}{\sqrt{1+x}} dx = \left| \begin{array}{l} t = 1+x \quad dt = dx \\ x = -1 \rightarrow t = 0 \\ x = 1 \rightarrow t = 2 \end{array} \right|$$

$$= \sqrt{2} \int_0^2 t^{-\frac{1}{2}} dt = \sqrt{2} \left[2\sqrt{t} \right]_0^2 = \underline{\underline{4}}$$

$$(5) \quad \varphi(t) = (\underbrace{\cos t + t \sin t}_{\varphi_1(t)}, \underbrace{\sin t - t \cos t}_{\varphi_2(t)}) \quad t \in [0, 2\pi]$$

$$\underline{\underline{\ell(\varphi) = \int_0^{2\pi} |\varphi'(t)| dt = \int_0^{2\pi} \sqrt{(\varphi_1'(t))^2 + (\varphi_2'(t))^2} dt}}$$

$$\uparrow (\varphi_3'(t))^2 \quad d=3$$

$$\varphi(t) = (t, f(t)) \quad \varphi_1(t) = t \quad \varphi_1'(t) = \boxed{1}$$

$$\varphi_1(t) = \underline{\cos t} + \underline{t \sin t} \quad \varphi_2(t) = \underline{\sin t} - \underline{t \cos t} \quad t \in (0, 2\pi]$$

$$\varphi_1'(t) = \cancel{-\sin t} + \cancel{\sin t} + \underline{t \cos t} = t \cos t$$

$$\varphi_2'(t) = \cancel{\cos t} - \cancel{\cos t} + \underline{t \sin t} = t \sin t$$

$$\int_0^{2\pi}$$

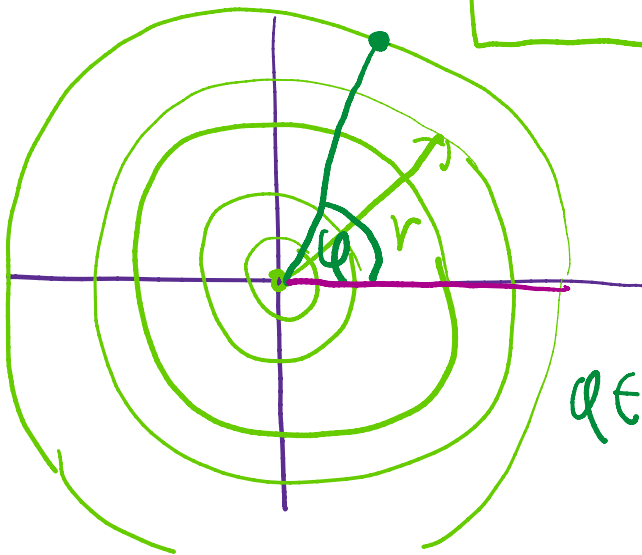
$$l(\varphi) = \int_0^{2\pi} \sqrt{(\underbrace{t \cos t}_{\text{blue}})^2 + (\underbrace{t \sin t}_{\text{blue}})^2} dt$$

$$= \int_0^{2\pi} t \underbrace{\sqrt{\cos^2 t + \sin^2 t}}_{=1} dt = \int_0^{2\pi} t dt = \left[\frac{t^2}{2} \right]_0^{2\pi} = \underline{\underline{2\pi^2}}$$

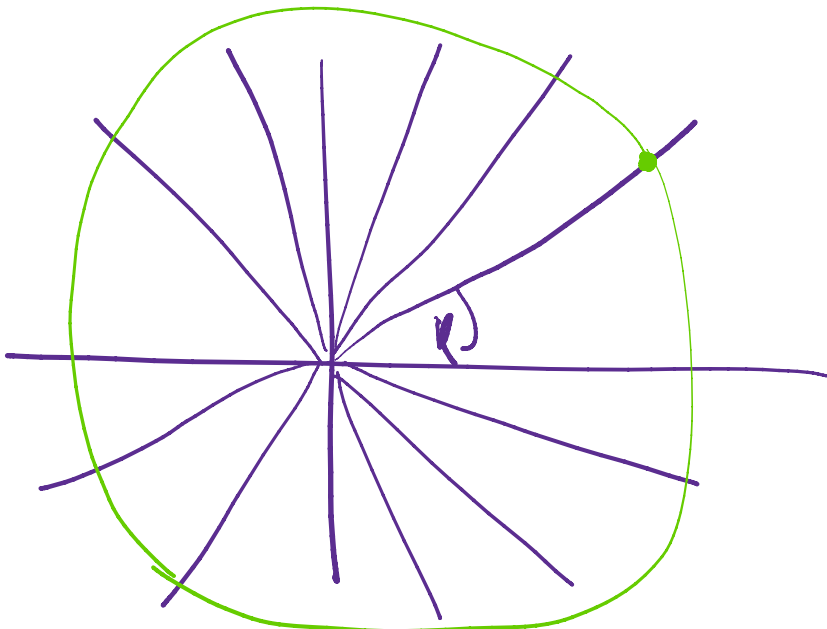
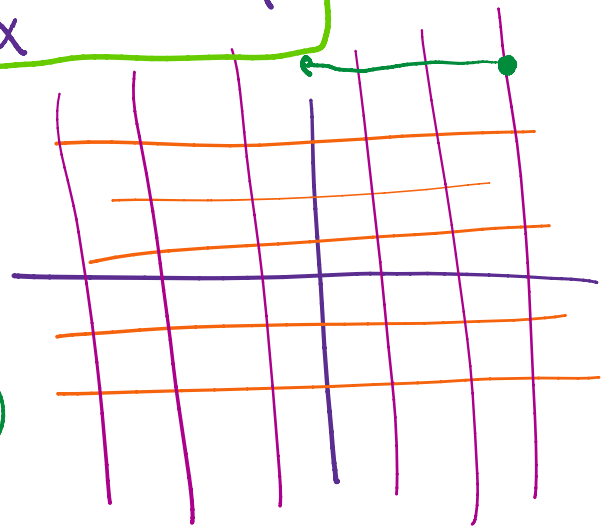
(8)

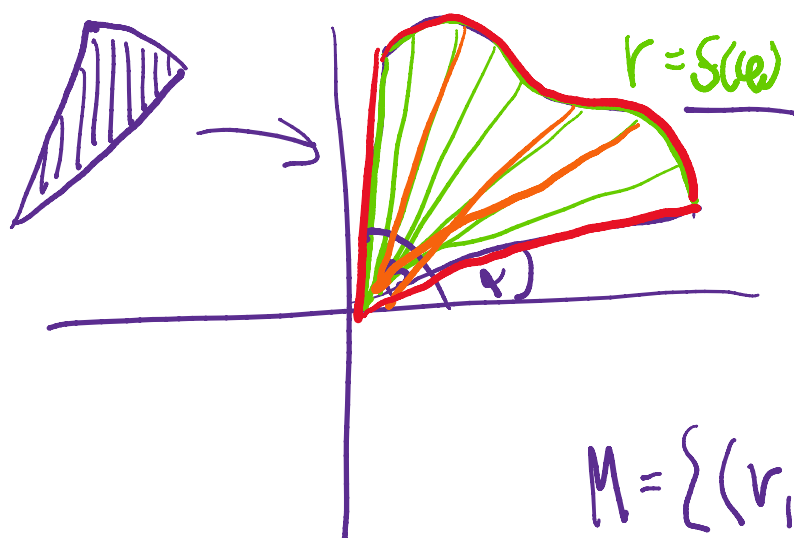
1c

$$S = \frac{1}{2} \int_{\alpha}^{\beta} f^2(\varphi) d\varphi$$

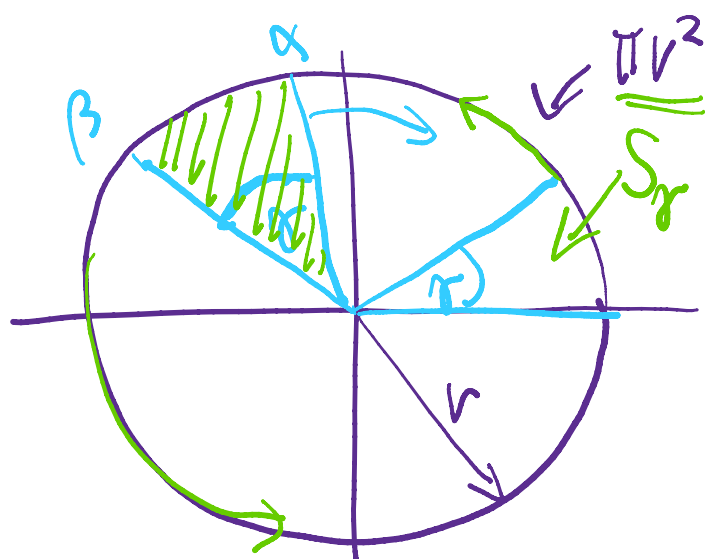
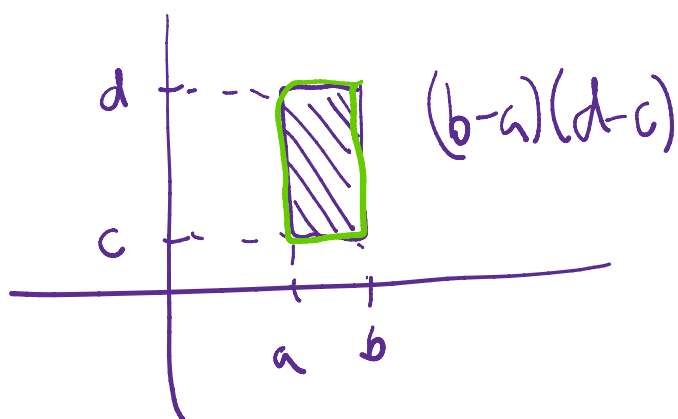


$\varphi \in [0, 2\pi)$





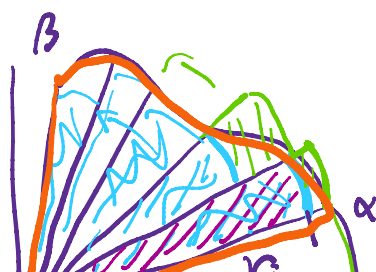
$$M = \{(r, \varphi) : \alpha \leq \varphi \leq \beta, 0 < r \leq S(\varphi)\}$$



$$\varphi = [0, 2\pi]$$

$$S_\varphi = \frac{\varphi}{2} r^2$$

$$= \frac{1}{2} (\beta - \alpha) r^2$$



$$D = \mathcal{D}([\alpha, \beta])$$



$$\alpha = \gamma_0 < \gamma_1 < \dots < \gamma_n = \beta$$

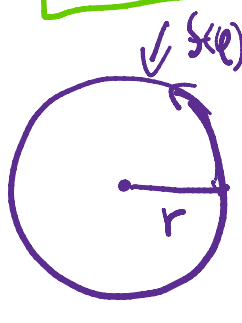
$$s(D) = \sum \frac{1}{2} (\gamma_{i+1} - \gamma_i) r_i^2 = \sum \underline{\frac{1}{2} r_i^2} (\gamma_{i+1} - \gamma_i)$$

$$S(D) = \sum \frac{1}{2} (\gamma_{i+1} - \gamma_i) R_i^2 = \sum \underline{\frac{1}{2} R_i^2} (\gamma_{i+1} - \gamma_i)$$

$$g(\varphi) = \underline{\frac{1}{2} f'(\varphi)^2}$$

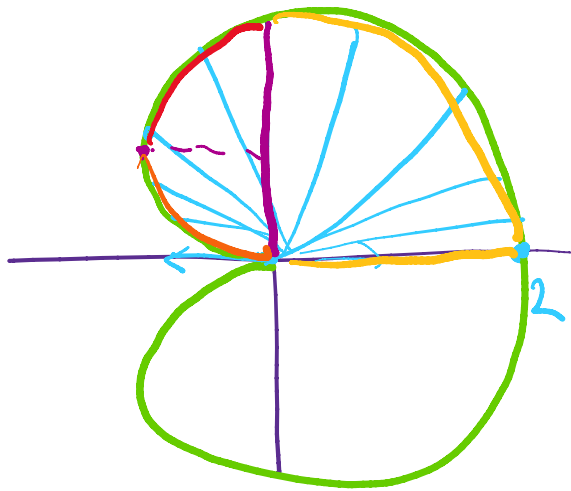
$$S(D) = S(g, D) \quad S(D) = \underline{S(g, D)}$$

$$S = \int_{\alpha}^{\beta} g(\varphi) d\varphi = \frac{1}{2} \int_{\alpha}^{\beta} f'(\varphi)^2 d\varphi$$



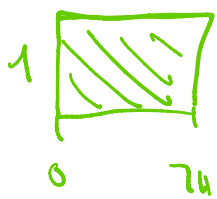
$$\rightarrow \frac{1}{2} \int_0^{2\pi} r^2 d\varphi = \underline{\frac{1}{2}} \left[r^2 \varphi \right]_{\varphi=0}^{2\pi} = \underline{\underline{\pi r^2}}$$

(8) $\underline{r(\alpha)} = \underline{1 + \cos \alpha} \quad \alpha \in [0, 2\pi]$

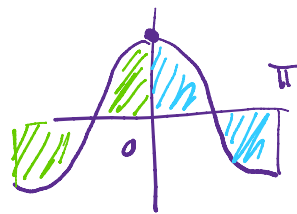


$$S = \frac{1}{2} \int_0^{2\pi} r^2(\alpha) d\alpha = \frac{1}{2} \int_0^{2\pi} (1 + \cos \alpha)^2 d\alpha$$

$$= \frac{1}{2} \int_0^{2\pi} 1 + 2\cos \alpha + \cos^2 \alpha d\alpha$$

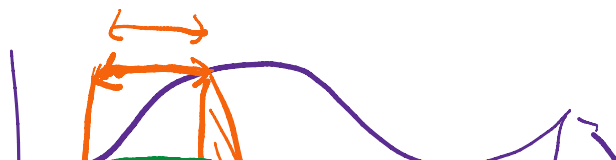


$$= \frac{1}{2} \left(\int_0^{2\pi} 1 d\alpha + 2 \int_0^{2\pi} \cos \alpha d\alpha + \int_0^{2\pi} \cos^2 \alpha d\alpha \right)$$

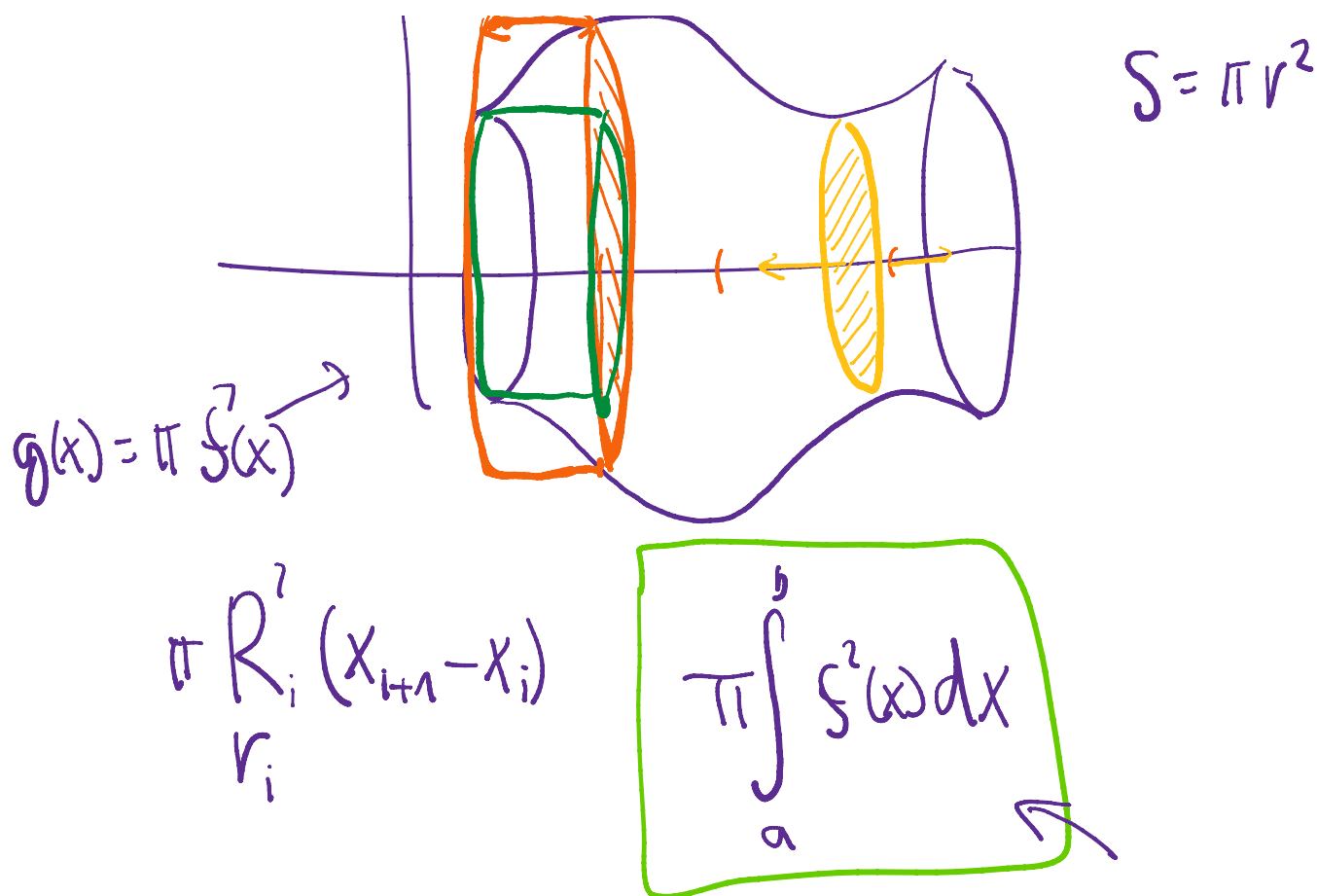


$$\cos^2 \alpha + \sin^2 \alpha = 1$$

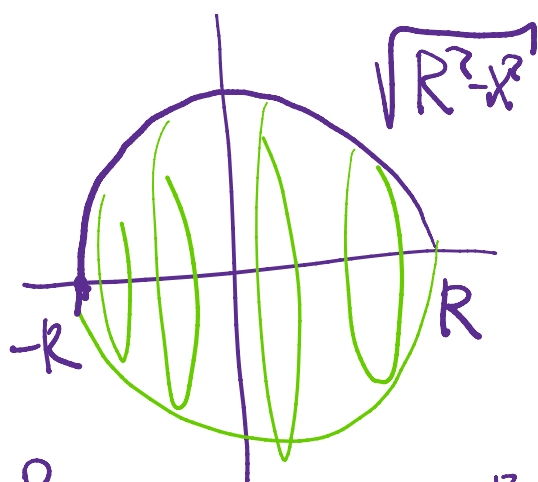
$$= \frac{1}{2} \left(\underline{2\pi} + 0 + \pi \right) = \underline{\underline{\frac{3\pi}{2}}}$$



$$S = \pi r^2$$



(a) Objem a povrch koule s poloměrem $R > 0$.

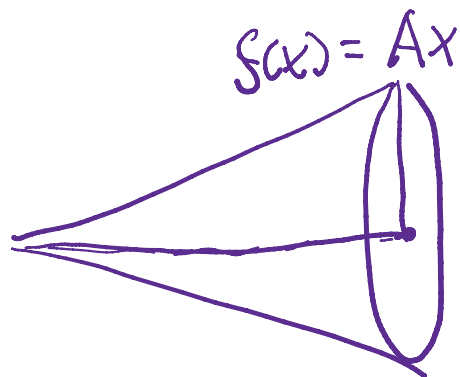


$$V = \pi \int_{-R}^R (\sqrt{R^2 - x^2})^2 dx = \pi \int_{-R}^R (R^2 - x^2) dx = \pi \left[R^2 x - \frac{x^3}{3} \right]_{-R}^R$$

1 2 3 4

$$= \pi \left[R^3 - \frac{R^3}{3} - \left(-R^3 + \frac{R^3}{3} \right) \right]$$

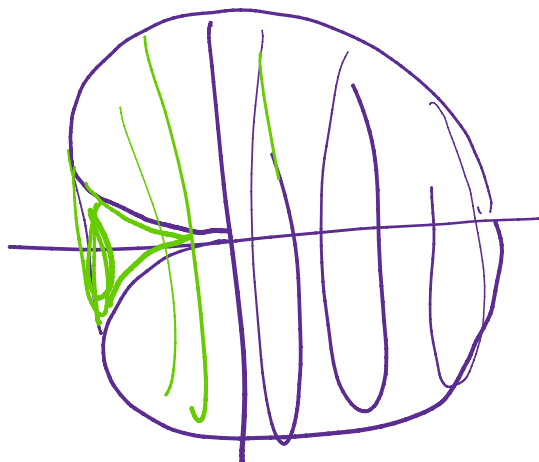
$$= \pi \left[2R^3 - \frac{2R^3}{3} \right] = \underline{\underline{\frac{4}{3} \pi R^3}}$$



(4b)

$$\bullet S = 2\pi \int_a^b S(x) \cdot \sqrt{1 + (S'(x))^2} dx$$

(10) $r(\alpha) = 1 + \cos \alpha$ $\alpha \in [0, \pi]$



B

$$V = \frac{2}{3}\pi \int_{\alpha}^{\beta} f^3(\alpha) \sin \alpha \, d\alpha$$

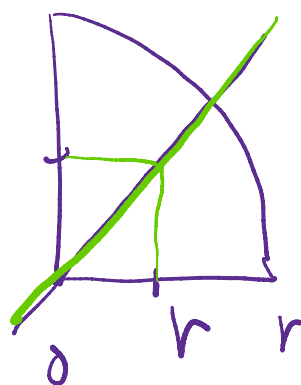
$$= \frac{2}{3}\pi \int_0^{\pi} (1 + \cos \alpha)^3 \cdot \sin \alpha \, d\alpha$$

$$= \left| \begin{array}{l} t = \cos \alpha + 1, dt = -\sin \alpha \, d\alpha \\ \alpha = 0 \rightarrow t = 2 \\ \alpha = \pi \rightarrow t = 0 \end{array} \right|$$

$$= \frac{2}{3}\pi \int_2^0 -t^3 \, dt = \frac{2}{3}\pi \int_0^2 t^3 \, dt$$

$$= \frac{2}{3}\pi \left[\frac{t^4}{4} \right]_0^2 = \frac{2}{3} \cdot \pi \cdot 4 = \underline{\underline{\frac{8}{3}\pi}}$$

(11)



$r(x)$

$$g(x) = \sqrt{r^2 - x^2}$$

$$\begin{aligned} \underline{M_x} &= \frac{1}{2} \int_0^r S(x) dx = \frac{1}{2} \int_0^r (r^2 - x^2) dx \\ &= \frac{1}{2} \left[r^2 x - \frac{x^3}{3} \right]_0^r = \frac{1}{2} \cdot \frac{2}{3} r^3 = \underline{\underline{\frac{1}{3} r^3}} \end{aligned}$$

$$\underline{M_y} = \int_0^r x f(x) dx = \frac{1}{2} \int_0^r x \sqrt{r^2 - x^2} dx$$

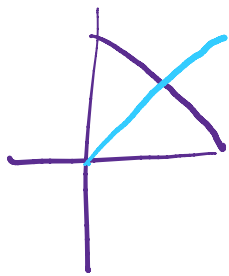
$$= \left| \begin{array}{l} t = r^2 - x^2 \quad dt = -2x dx \\ x=0 \rightarrow t=r^2 \\ x=r \rightarrow t=0 \end{array} \right|$$

$$= -\frac{1}{2} \int_{r^2}^0 \sqrt{t} dt = \frac{1}{2} \int_0^{r^2} t^{\frac{1}{2}} dt = \frac{1}{2} \left[\frac{2}{3} t^{\frac{3}{2}} \right]_0^{r^2}$$

$$= \underline{\underline{\frac{1}{3} r^3}}$$

$$M = \int_0^r f(x) dx = \int_0^r \sqrt{r^2 - x^2} dx = \underline{\underline{\frac{\pi r^2}{4}}}$$

$$= \left| x = r \sin t \quad dx = r \cos t dt \right|$$



$$= \left| \begin{array}{l} x = r \sin t \quad dx = r \cos t \, dt \\ r \rightarrow t = \frac{\pi}{2} \\ 0 \rightarrow t = 0 \end{array} \right|$$

$$= \int_0^{\frac{\pi}{2}} r \cdot \cos t \cdot r \cdot \cos t \, dt = r^2 \int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

$$= r^2 \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \underline{\underline{\frac{\pi r^2}{4}}}$$

$$T = \left(\frac{\frac{1}{3} r^3}{\frac{\pi r^2}{4}}, \frac{\frac{1}{3} r^3}{\frac{\pi r^2}{4}} \right) = \underline{\underline{\frac{4}{3} \cdot \frac{r}{\pi}}}$$