

$$\sum_{n=1}^{\infty} a_n \quad a_n > 0 \quad n \in \mathbb{N}$$

• nutná podmínka  $a_n \rightarrow 0$

$$(1) \quad \sum_{n=1}^{\infty} \frac{n^2}{n^3+1} = \infty$$

$$a_n \sim \frac{1}{n^\alpha} = b_n \quad \lim_{n \rightarrow \infty} \frac{b_n}{a_n} = 1$$

$$\bullet \quad \sum_{n=1}^{\infty} a_n < \infty \Leftrightarrow \sum_{n=1}^{\infty} b_n < \infty$$

$$\sum \frac{1}{n^\alpha} < \infty \Leftrightarrow \alpha > 1$$

$$\sum \frac{3^n + 2^n}{4^n + 5^n} < \infty$$

$\underline{\hspace{1.5cm}} = a_n$

$$\frac{3^n + 2^n}{4^n + 5^n} \ll \frac{2 \cdot 3^n}{5^n}$$

$$a_n \sim q^n \quad \left( \frac{3}{5} \right)^n$$

$$q \geq 0$$

$$\frac{\frac{3^n + 2^n}{4^n + 5^n}}{\left(\frac{3}{5}\right)^n} = \frac{1 + \left(\frac{2}{3}\right)^n}{\left(\frac{4}{5}\right)^n + 1} \rightarrow 1$$

$$\bullet \sum q^n < \infty \Leftrightarrow q < 1$$

$$(2) \bullet \sum_{n=1}^{\infty} \frac{n^7}{4^n} < \infty$$

$$\left( \frac{\frac{(n+1)^7}{4^{n+1}}}{\frac{n^7}{4^n}} = \frac{1}{4} \left( \frac{n+1}{n} \right)^7 \right)$$

$$b_n = \frac{1}{3^n}$$

$$\frac{\frac{1}{3^n}}{\frac{n^7}{4^n}} = \frac{\left(\frac{4}{3}\right)^n}{n^7} \rightarrow \infty$$

$$\lim_{n \rightarrow \infty} \frac{b_n}{a_n} = L > 0 \Rightarrow \sum b_n < \infty \Rightarrow \sum a_n < \infty$$

$$\sum \left(\frac{1}{3}\right)^n \quad \frac{1}{3} < 1$$

$$b_n = \frac{1}{4^n} \quad \frac{b_n}{a_n} = \frac{1}{n^7} \rightarrow 0 < \infty$$

$$\dots \quad \frac{1}{a_n} = n^2 \dots$$

$$\sum a_n < \infty \Rightarrow \sum b_n < \infty$$

$$(3) \quad \sum_{n=1}^{\infty} \frac{2^n}{n!} \quad a_n \rightarrow 0$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L \quad \begin{cases} L < 1 \Rightarrow \sum a_n < \infty \\ L > 1 \Rightarrow \sum a_n = \infty \end{cases}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \frac{2}{n+1} \rightarrow 0 < 1 \Rightarrow \sum a_n < \infty$$

$$(5) \quad \sum_{n=1}^{\infty} \left( \overset{a}{\downarrow} \sqrt{n^3+1} - \overset{b}{\downarrow} \sqrt{n^3-1} \right) \quad a_n$$

$$\begin{aligned} a^2 - b^2 &= (a+b)(a-b) \\ a-b &= \frac{a^2 - b^2}{a+b} \end{aligned}$$

$$a_n \rightarrow 0 ?$$

$$a_n = \frac{n^3+1 - (n^3-1)}{\sqrt{n^3+1} + \sqrt{n^3-1}} = \frac{2}{\sqrt{n^3+1} + \sqrt{n^3-1}} \rightarrow 0$$

$$\dots \sqrt{n^2+1} + \sqrt{n^2-1} \quad \sqrt{k^2+1} + \sqrt{k^2-1}$$

$$a_n \sim \frac{1}{n^\alpha} \quad \alpha = \frac{3}{2} > 1 \Rightarrow \sum_{n=1}^{\infty} a_n < \infty$$

$$(3) \quad \sqrt[n]{\frac{2^n}{n!}} = \frac{2}{\sqrt[n]{n!}} \rightarrow 0$$

$$(6) \quad \sum_{n=1}^{\infty} \frac{n!}{2^{(n^2)}} < \infty$$

$$\left| \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{2^{n^2+2n+1}}}{\frac{n!}{2^{n^2}}} = \frac{n+1}{2^{2n+1}} \rightarrow 0 < 1 \right.$$

$$(7) \quad \sum_{n=0}^{\infty} \frac{n!}{n^n} \quad a_n \rightarrow 0$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \left(\frac{n}{n+1}\right)^n \rightarrow \frac{1}{e} \left[ \frac{1}{\left(\frac{n+1}{n}\right)^n} \right]$$

$$\frac{1}{e} < 1 \Rightarrow \sum a_n < \infty$$

$$\left(1 + \frac{1}{n}\right)^n \rightarrow e$$

$$\frac{1}{e} < 1 \Rightarrow \sum_{n=1}^{\infty} a_n < \infty$$

$$\underline{(1 + \frac{1}{n}) \rightarrow e}$$

$$\underline{n\sqrt[n]{a_n} = \frac{\sqrt[n]{n!}}{n} \rightarrow \frac{1}{e}}$$

$$(8) \quad \sum_{n=1}^{\infty} \frac{4n + 2n}{(n + n + 1)^3} \stackrel{!}{=} a_n$$

$\sim n^4$

$$\lim_{n \rightarrow \infty} a_n = 4 \neq 0 \rightarrow \sum_{n=1}^{\infty} a_n = \infty$$

$$(4) \quad \sum_{n=1}^{\infty} \binom{2n}{n} \frac{1}{5^n} \stackrel{!}{=} a_n$$

$$\frac{a_{n+1}}{a_n} = \frac{\binom{2n+2}{n+1} \cdot \frac{1}{5^{n+1}}}{\binom{2n}{n} \cdot \frac{1}{5^n}} = \frac{1}{5} \frac{(2n+2)!}{[(n+1)!]^2} \frac{(n!)^2}{(2n)!}$$

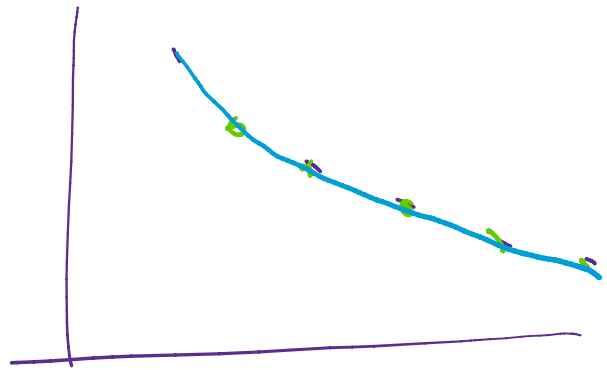
$\nearrow 4n^2$

$$= \frac{1}{5} \frac{(2n+2)(2n+1)}{(n+1)^2} \rightarrow \frac{4}{5} < 1 \quad \left[ \left( \frac{4}{5} \right)^n \right]$$

$$= \frac{1}{5} \frac{1}{(n+1)^2} \rightarrow \frac{1}{5} < 1 \quad | (5) |$$

$$\sum a_n < \infty$$

$$\sum_{n=2}^{\infty} \frac{1}{n \log^2 n}$$



$$\sum_{n=2}^{\infty} f(n) < \infty \Leftrightarrow \int_2^{\infty} f(x) < \infty$$

$$\int_2^{\infty} \frac{1}{x \log^2 x} \quad \left| \begin{array}{l} t = \log x \\ dt = \frac{1}{x} dx \end{array} \right| = \int_{\log 2}^{\infty} \frac{1}{t^2} < \infty$$

$$\bullet \sum \frac{1}{n \log^{\alpha} n} < \infty \Leftrightarrow \alpha > 1$$

$$\sum_{n=1}^{\infty} a_n < \infty \quad \bullet \quad a_n \text{ nevostonci!} \quad \sum_{n=1}^{\infty} 2^n a_n$$



$$\sum_{n=1}^{\infty} a_n < \infty \quad \sum_{n=1}^{\infty} \underline{2^n} a_{2^n} \quad \dots \dots \dots$$

$$a_n = \frac{1}{n \log 2 n} \quad a_{2^n} = \frac{1}{\cancel{2^n} \cdot \log^2(2^n)}$$

$$\sum_{n=1}^{\infty} \cancel{2^n} a_{2^n} = \sum_{n=1}^{\infty} \frac{1}{n \log^2(2)}$$

$\uparrow$   
 $\frac{1}{n^2} \rightarrow q^n$