Cvičení 19.3.

pátek 19. března 2021 11:30

(1) $y' = 1-y \quad \rightarrow \quad f(x) = 1 \quad g(y) = 1-y$

$$y' = f(x)\cdot g(y)$$

(1) maximální intervaly v D_f $\rightarrow D_f = \mathbb{R} \rightarrow I = \mathbb{R}$

(2) max. intervaly v D_g po odebrání nulových bodů g

$D_g = \mathbb{R}$, $g(y) = 0 \Leftrightarrow y = 1 \rightarrow J_1 = (-\infty, 1)$, $J_2 = (1, \infty)$

(3) stacionární řešení (nulové body g) $\rightarrow y_1(x) = 1$, $x \in I$

(4) $F(x) \stackrel{c}{=} \int f = x \rightarrow F(x) = x$, $x \in \mathbb{R}$

$$\int \frac{1}{g} = \int \frac{1}{1-y}\,dy \stackrel{c}{=} -\log|1-y|$$

- $J_1 \rightarrow G(y) = -\log(1-y)$
- $J_2 \rightarrow G(y) = -\log(y-1)$

$D_{G^{-1}} = G(J_1)$

(5) I, J_1, $C \in \mathbb{R}$, $\{x \in I : F(x) + C \in G(J_1)\} = \mathbb{R}$

$$G(J_1) = G((-\infty, 1)) = (-\infty, \infty)$$

$$\lim_{y \to -\infty} G(y) = -\infty \qquad \lim_{y \to 1-} G(y) = \infty$$

I^*

$$I^* = \mathbb{R} \rightarrow \boxed{y_2(x) = G^{-1}(F(x)+C) = 1 - e^{-(x+C)}, \; x \in \mathbb{R}}$$

$$t = -\log(1-y) \rightarrow e^{-t} = 1-y \rightarrow \boxed{y = 1 - e^{-t}} = G^{-1}(t)$$

I, J, C, $\{x \in I : F(x) + C \in G(J)\} = \mathbb{R} = I^*$

$$G^{-1}(t)$$

$I, J_2, C \quad \{x \in I : F(x) + C \in G(J_2)\} = \mathbb{R} = I^*$

$G(y) = -\log(y-1) \qquad y \in (1, \infty) = J_2$

$\lim\limits_{y \to 1+} G(y) = +\infty \qquad \lim\limits_{y \to \infty} G(y) = -\infty$

$G(J_2) = \mathbb{R}$

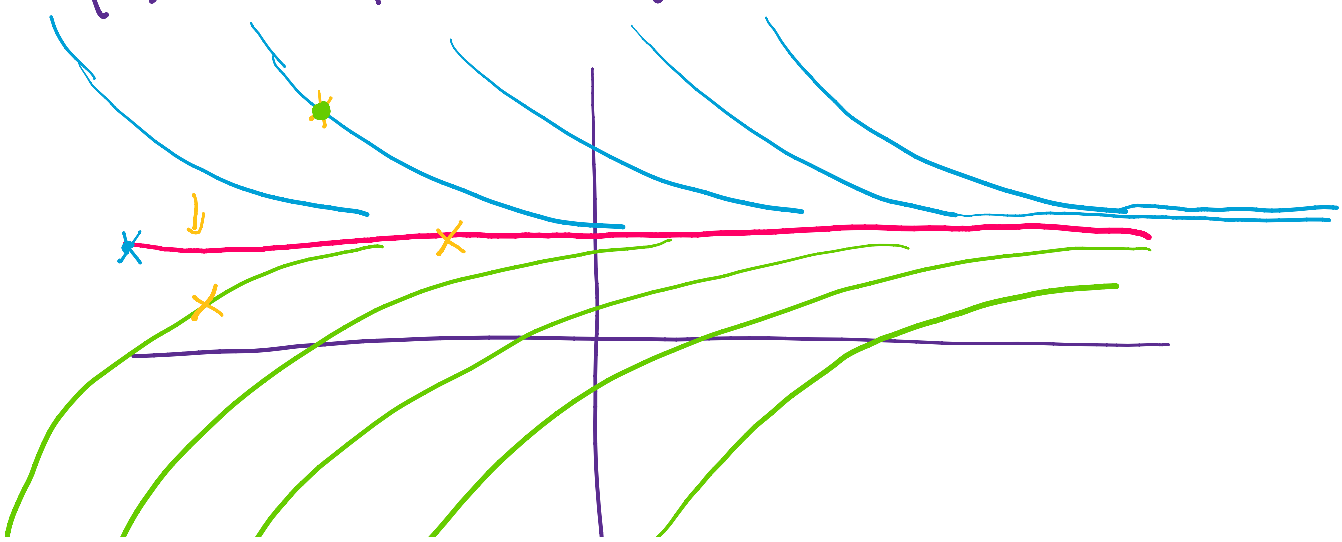
$y_3(x) = G^{-1}(F(x) + C) = \boxed{1 + e^{-(x+C)}, \quad x \in \mathbb{R}}$

$t = -\log(y-1) \qquad y - 1 = e^{-t} \qquad y = 1 + e^{-t}$

$$\left.\begin{aligned} &y_1(x) = 1 = 1 + 0 \cdot e^{-x} \\ &y_2(x) = 1 - e^{-(x+C)} = 1 - De^{-x} \\ &y_3(x) = 1 + e^{-(x+C)} = 1 + De^{-x} \end{aligned}\right\} x \in \mathbb{R} \qquad \overset{e^{-C}}{\overset{\shortparallel}{D}} > 0$$

$y(x) = 1 + De^{-x} \qquad x \in \mathbb{R}, \; D \in \mathbb{R}$

(6) nelze lepit, řešení jsou maximální



(2) $y' = y^2 - 4,$

(3) $y' = 2x^2 y^2 \quad \to \quad f(x) = 2x^2 \quad g(y) = y^2$

(1) $D_f = \mathbb{R} \qquad I = \mathbb{R}$

(2) $D_g = \mathbb{R}, \quad g(y) = 0 \iff y = 0 \quad \to \quad \underbrace{(-\infty, 0)}_{J_1 :=}, \ \underbrace{(0, \infty)}_{=: J_2}$

(3) $y_1(x) = 0 \quad , x \in I = \mathbb{R}$

(4) $F(x) = \frac{2}{3} x^3 \qquad x \in \mathbb{R}$

$G(y) \overset{c}{=} \int \frac{1}{y^2} dy \overset{c}{=} -\frac{1}{y} \quad \to J_1 \to G(y) = -\frac{1}{y}, \ y \in (-\infty, 0)$

$J_2 \to G(y) = -\frac{1}{y}, \ y \in (0, \infty)$

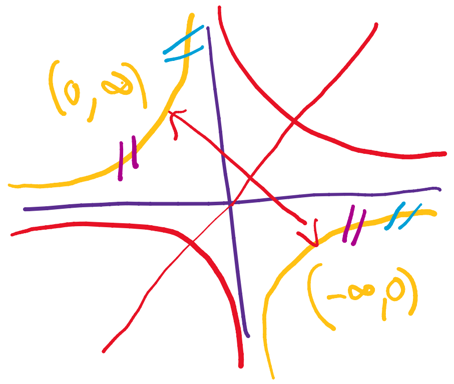
(5) $I, J_1, C \in \mathbb{R}, \quad \{x \in \mathbb{R}, \ F(x) + C \in G(J_1)\}$

$G(J_1) = (0, \infty)$

- $G^{-1}(t) = -\frac{1}{t} \qquad t \in (0, \infty)$

- $F(x) + C \in (0, \infty) \iff \frac{2}{3} x^3 + C \in (0, \infty)$

$\iff \frac{2}{3} x^3 \in (-C, \infty) \iff x^3 \in \left(-\frac{3}{2} C, \infty\right)$



$$\iff \frac{2}{3}x^3 \in (-C,\infty) \iff x^3 \in \left(-\frac{3}{2}C,\infty\right)$$

$$\iff x \in \left(-\sqrt[3]{\frac{3}{2}C}, \infty\right) = I^*$$

$$\boxed{y_2(x) = G^{-1}(F(x)+C) = -\frac{1}{\frac{2}{3}x^3+C} \qquad x\in\left(-\sqrt[3]{\frac{3}{2}C},\infty\right)}$$

$I, J_2, C \qquad \{x\in\mathbb{R} : F(x)+C \in G(J_2)\}$

$$\frac{2}{3}x^3 + C \in (-\infty,0)$$

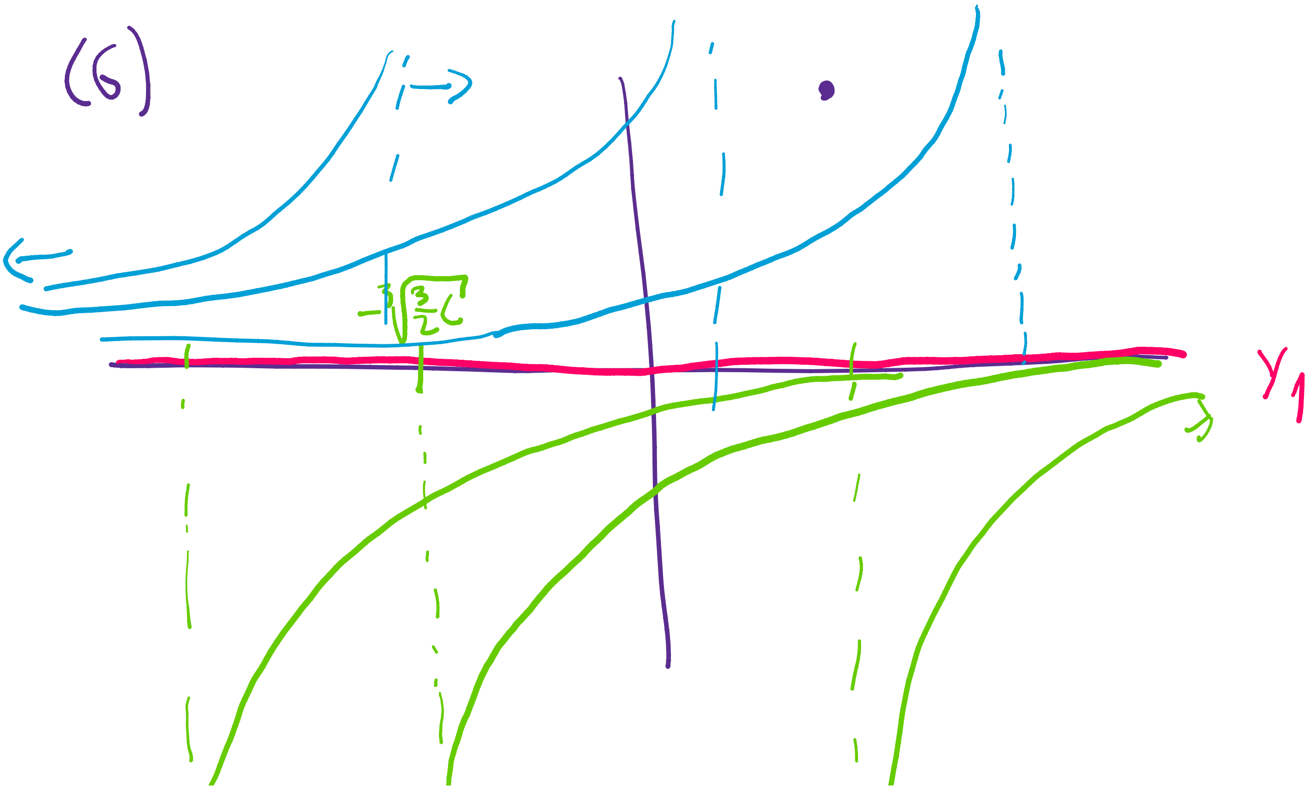
$$G^{-1}(t) = -\frac{1}{t} \quad t\in(-\infty,0)$$

$$\frac{2}{3}x^3+C\in(-\infty,0)\iff \frac{2}{3}x^3\in(-\infty,-C)\iff x^3\in\left(-\infty,-\frac{3}{2}C\right)$$

$$x\in\left(-\infty,-\sqrt[3]{\frac{3}{2}C}\right) = I^*$$

$$\boxed{y_3(x) = -\frac{1}{\frac{2}{3}x^3+C}, \quad x\in\left(-\infty,-\sqrt[3]{\frac{3}{2}C}\right)}$$

(6)



lepit nelze, řešení jsou maximální ↓

(4) $y' = y^{\frac{2}{3}}$ $\quad f(x)=1 \quad g(y)=y^{\frac{2}{3}}$

(1) $D_f=\mathbb{R} \quad I=\mathbb{R}$

(2) $D_g=\mathbb{R}$, $g(y)=0 \iff y=0 \quad J_1=(-\infty,0), J_2=(0,\infty)$

(3) $y_1(x)=0$, $x\in\mathbb{R}$.

(4) $F(x)=x \quad \swarrow y^{-\frac{2}{3}}$

• $G(y) \stackrel{c}{=} \int \frac{1}{y^{\frac{2}{3}}}\,dy \stackrel{c}{=} 3y^{\frac{1}{3}}$

$G(y)=3y^{\frac{1}{3}} \quad y\in(-\infty,0)$

$=3y^{\frac{1}{3}} \quad y(0,\infty)$

(5) $I, J_1, C \quad \{x\in\mathbb{R}: \underline{x+C\in(-\infty,0)}\}$

$G((-\infty,0))=(-\infty,0)$

$x+C\in(-\infty,0) \iff x\in(-\infty,-C)=I^*$

• $\boxed{y_2(x)=G^{-1}(F(x)+C)=\frac{1}{27}(x+C)^3, \quad x\in(-\infty,-C)}$

$-C$

$G^{-1}(t)=\frac{1}{27}t^3$

$-C$

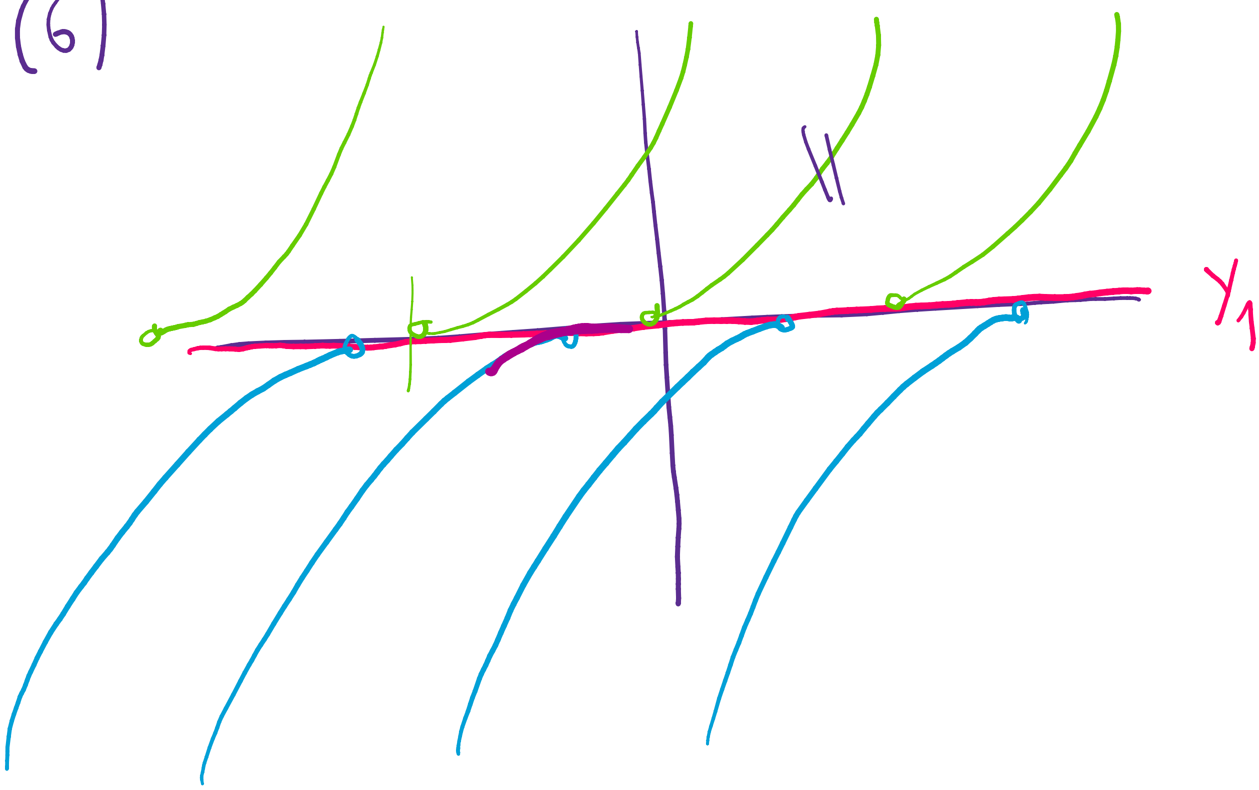
$$G^{-1}(t) = \frac{1}{27}t^3$$

$I_1, J_2, C \quad \{x \in \mathbb{R} : x + C \in (0,\infty)\}$

$x + C \in (0,\infty) \iff x \in (-C,\infty) = I^*$

- $$\boxed{y_3(x) = \frac{1}{27}(x+C)^3, \quad x \in (-C,\infty)}$$

(6)



$$y_1(x) = 0$$

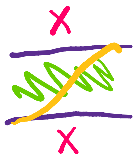
- $$\tilde{y}_1(x) = \begin{cases} 0, & x \in (-\infty, -C], \\ \frac{1}{27}(x+C)^3, & x \in (-C, \infty). \end{cases}$$

- $$\tilde{y}_2(x) = \begin{cases} \frac{1}{27}(x+C)^3 & x \in (-\infty, -C), \\ 0 & x \in [-C, \infty). \end{cases}$$

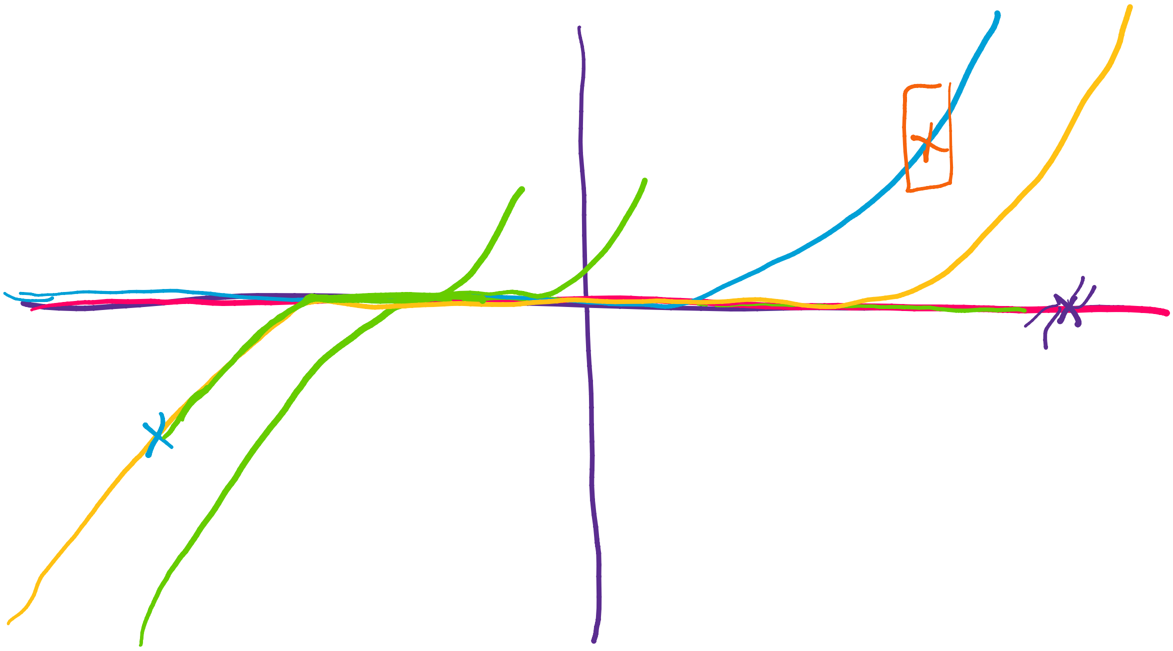
$0 \qquad x \in [-C, \infty).$

$$\tilde{y}_3(x) = \begin{cases} \frac{1}{27}(x+C)^3 & x \in (-\infty, -C) \\ 0 & x \in [-C, -D] \\ \frac{1}{27}(x+D)^3 & x \in (-D, \infty) \end{cases} \qquad -C \leq -D$$

podobně (5) - (7)



$\sqrt{y-x^2}$



(8) $\bullet\ xy' + y = y^2$

$x \neq 0$ — ekvivalentní rovnice $\boxed{y' = \frac{y^2 - y}{x}}$

$f(x) = \frac{1}{x} \qquad g(y) = y^2 - y$

(1) $D_f = \mathbb{R} \setminus \{0\} \qquad I_1 = (-\infty, 0),\ I_2 = (0, \infty)$

(2) $D_g = \mathbb{R},\quad g(y) = 0 \iff y = 0, 1$

(2) $y_g = \mathbb{R}, \quad g(y) = 0 \iff y = 0, 1.$

$J_1 = (-\infty, 0), \; J_2 = (0, 1), \; J_3 = (1, \infty)$

(3) $y_1(x) = 0 \qquad x \in I_1, I_2$

$y_2(x) = 1 \qquad x \in I_1, I_2$

(4) $F(x) \overset{c}{=} \int \frac{1}{x} = \log|x|$

- $F(x) = \log(-x), \; x \in I_1$

  $F(x) = \log(x), \; x \in I_2$

$$G(y) = \int \frac{1}{y^2 - y} dy = \int \frac{1}{y-1} - \frac{1}{y} dy$$

$$\overset{c}{=} \log|y-1| - \log|y| = \log\left|\frac{y-1}{y}\right|$$

- $G(y) = \log\left(\frac{y-1}{y}\right), \quad y \in J_1 = (-\infty, 0) \qquad G(J_1) = G(-\infty, 0)$

- $G(y) = \log\left(\frac{1-y}{y}\right), \quad y \in J_2 = (0, 1) \to (-\infty, +\infty) \qquad (0, +\infty)$

  $G(y) = \log\left(\frac{y-1}{y}\right), \quad y \in J_3 = (1, \infty) \qquad G(J_3) = (-\infty, 0)$

$-\infty, 0$

(5) $I_1, J_3, C \quad \{x \in (-\infty, 0) : \log(-x) + C \in (-\infty, 0)\}$

$\log(-x) + C \in (-\infty, 0) \iff \log(-x) \in (-\infty, -C)$ $\qquad (0, \infty)$

$\iff -x \in (0, e^{-C}) \iff x \in (-e^{-C}, 0) = I^*$

$y(x) = G^{-1}(F(x) + C) = \frac{1}{\;\cdots\;} = \frac{1}{\;\cdots\;}, \; x \in I^*$

$$y_3(x) = G^{-1}(F(x)+c) = \frac{1}{1-e^{\log(-x)+c}} = \frac{1}{1+xe^c}, \quad x \in I^*$$

$$t = \log\frac{y-1}{y} \iff e^t = \frac{y-1}{y} \iff ye^t = y-1$$

$$\iff y(e^t-1) = -1 \iff y(x) = \frac{1}{1-e^t}$$

$I_1, J_2, C, \quad \overset{I_1}{\{x \in (-\infty,0) : \log(-x)+C \in \underbrace{G((0,1))}_{\mathbb{R}}\}}$

$G((0,1)) = (-\infty,\infty) \qquad I^* = I$

$$\boxed{y_4(x) = G^{-1}(F(x)+C) = \frac{1}{e^{\log(-x)+c}+1} = \frac{1}{1-e^c \cdot x}, \quad x \in (-\infty,0)}$$

$G(y) = \log\left(\frac{1-y}{y}\right) \qquad t = \log\left(\frac{1-y}{y}\right)$

$e^t = \frac{1-y}{y} \qquad y(e^t+1) = 1$

$G^{-1}(t) = \frac{1}{e^t+1}$

$I, J_1, C,$

$$\boxed{y_5(x) = \frac{1}{1+xe^c}, \quad x \in (-\infty, -e^c)}$$