Cvičení 19.5.

středa 19. května 2021 12:21

$$\boxed{xy^3 + y = 1}$$

$\{(x,y):\ f(x,y) = 0\}$

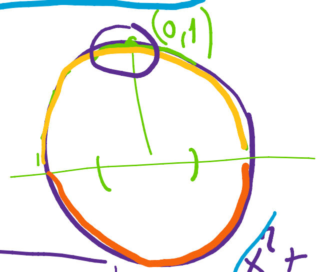
?

$$\boxed{f(x,y) = x^2 + y^2 - 1}$$

$$\boxed{y = \sqrt{1-x^2}} = \varphi(x)$$

• $f(x,y) = xy^3 + y - 1$

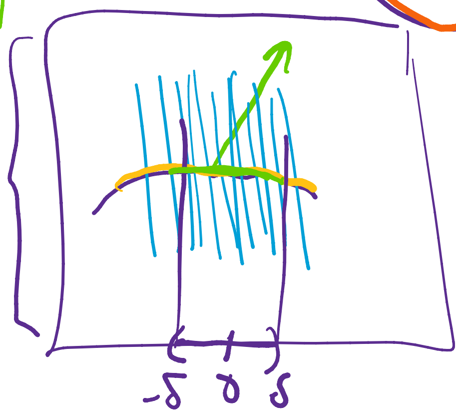
$(x,y) = \boxed{(0,1)}$



$\cancel{x^2} + \cancel{1} - \cancel{x^2} - \cancel{1} = 0$

$x^2 + (\varphi(x))^2 - 1 = 0$

$$\frac{\partial f}{\partial y}(0,1) \neq 0$$



$$\varphi : (-\delta, \delta) \to \mathbb{R}$$

$$f(x, \varphi(x)) = 0 \qquad x \in (-\delta, \delta)$$

$$f \in C^{1^k} \to \varphi \in C^{1^k}$$

$f(x,y) = xy^3 + y - 1$

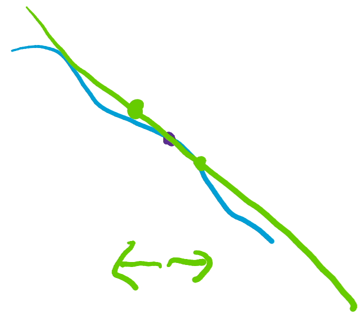
$$\frac{\partial f}{\partial y}(x,y) = 3xy^2 + 1 \qquad \frac{\partial f}{\partial y}(0,1) = 1 \neq 0 \qquad \boxed{\frac{\partial f}{\partial x} = y^3}$$

• $f(x, \varphi(x)) = 0$

1. $\frac{\partial f}{\partial x}(x, \varphi(x)) + \boxed{\varphi'(x)}\, \frac{\partial f}{\partial y}(x, \varphi(x)) = 0$

$\boxed{\partial f}\,(x, \varphi(x))$

- $$\boxed{\varphi'(x) = - \frac{\boxed{\frac{\partial f}{\partial x}}(x, \varphi(x))}{\boxed{\frac{\partial f}{\partial y}}(x, \varphi(x))}}$$ $x \in (-\delta, \delta)$



- $$\varphi'(0) = - \frac{\frac{\partial f}{\partial x}(0, 1)}{\frac{\partial f}{\partial x}(0, 1)} = - \frac{1}{1} = -1$$

$$0 = x\varphi(x)^3 + \varphi(x) - 1$$

$$\Rightarrow \boxed{0 = \varphi(x)^3 + x \cdot 3 \cdot \varphi(x)^2 \cdot \varphi'(x) + \varphi'(x)}$$

$$0 = \varphi'(x)\left(3x\varphi(x)^2 + 1\right) + \varphi(x)^3$$

$$\boxed{\varphi'(x) = - \frac{\varphi(x)^3}{3x\varphi(x)^2 + 1}}$$

$$\varphi'(0) = - \frac{\overset{1=}{\varphi(0)^3}}{3 \cdot 0 \cdot \varphi(0)^2 + 1} = -1$$

$$0 = \varphi(x)^3 + x \cdot 3 \cdot \varphi(x)^2 \cdot \varphi'(x) + \varphi'(x)$$

$$0 = 3\varphi(x)^2 \cdot \varphi'(x) + 3\varphi(x)^2 \cdot \varphi'(x) + 6x(\varphi'(x))^2 + 3x\varphi(x)^2\varphi''(x) + \varphi''(x)$$

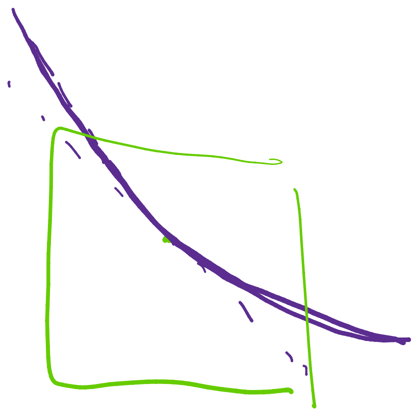
$+\varphi''(x)$

pro $x=0$

$$0 = 6\,\underbrace{\varphi(0)^2}_{1}\cdot\underbrace{\varphi'(0)}_{-1} + 6\cdot 0\cdot(\varphi'(0))^2 + 3\cdot 0\,\varphi(0)^2\cdot\varphi''(0) + \varphi''(0)$$

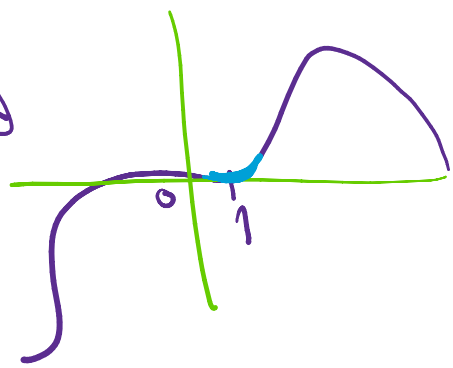
$$0 = -6 + \varphi''(0) \longrightarrow \boxed{\varphi''(0) = 6}$$

$$\boxed{\varphi(x) = 1 - x + 3x^2 + o(x^2)}$$



(4) $2x^4y + x^3 + y^3 + xy = 1 \qquad (1,0)$

$\boxed{\varphi(1)=0}$



$$f(x,y) = 2x^4y + x^3 + y^3 + xy - 1$$

$$\boxed{f(x,\varphi(x)) = 0, \quad \varphi(1)=0}$$

$$f(1,0) = 2\cdot 1\cdot 0 + 1 + 0 + 1\cdot 0 - 1 = 0$$

$$\frac{\partial f}{\partial y} = 2x^4 + 3y^2 + x \qquad \frac{\partial f}{\partial y}(1,0) = 2 + 3\cdot 0 + 1 = \boxed{3}$$

$$\frac{\partial f}{\partial x} = 8x^3y + 3x^2 + y \qquad \frac{\partial f}{\partial x}(1,0) = \boxed{3}$$

$$\boxed{\varphi'(1) = -\frac{3}{3} = -1}$$

$$\varphi(1) = 0$$

$$2x^4\varphi(x) + x^3 + \varphi(x)^3 + x\varphi(x) - 1 = 0$$

$$8x^3\varphi(x) + 2x^4\varphi'(x) + 3x^2 + 3\varphi(x)^2\varphi'(x) + \varphi(x) + x\varphi'(x) = 0$$

$x = 1$

$$8\cdot 1^3\varphi(1) + 2\cdot 1^4\cdot\varphi'(1) + 3\cdot 1^2 + 3\cdot 0^2\cdot\varphi'(1) + 0 + 1\cdot\varphi'(1) = 0$$

$$0 \qquad 3 + 3\varphi'(1) = 0 \to \boxed{\varphi'(1) = -1}$$

$$8x^3\varphi(x) + 2x^4\varphi'(x) + 3x^2 + 3\varphi(x)^2\varphi'(x) + \varphi(x) + x\varphi'(x) = 0$$

$$24x^2\varphi(x) + 8x^3\varphi'(x) + 8x^3\varphi'(x) + 2x^4\varphi''(x) + 6x +$$

$$6\varphi(x)(\varphi'(x))^2 + 3\varphi(x)^2\varphi''(x) + \varphi'(x) + \varphi'(x) + x\varphi''(x) = 0$$

$x = 1,\ \varphi(1) = 0,\ \varphi'(1) = -1$

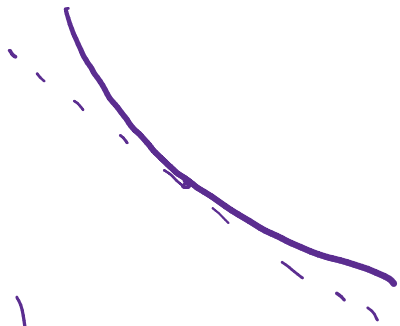
$$24\cdot 1\cdot 0 + 16(-1) + 2\varphi''(1) + 6 + 6\cdot 0(\varphi'(1))^2$$

$$24\cdot 1\cdot 0 + 16(-1) + 24(1) + 6 + 8\cdot 0\cdot(\varphi(1))$$

$$+3\cdot 0^2\cdot\varphi''(1) + (-1) + (-1) + 1\cdot\varphi''(1) = 0$$

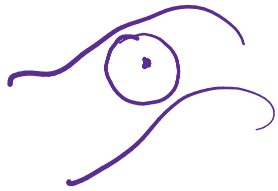
$$-16-2+6+3\varphi''(1) = 0$$

$$3\varphi''(1) = 12 \qquad \boxed{\varphi''(1) = 4}$$

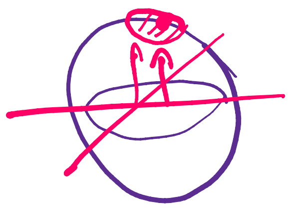


$$\left[\begin{array}{l} \varphi(1)=0,\ \varphi'(1)=-1,\ \varphi''(1)=4 \ \to \\ \varphi(x) = -(x-1)+2(x-1)^2 + o((x-1)^2) \end{array}\right.$$

(6) $x^2+y^2+z^2 = 3xyz$ $\qquad (1,1,1)$



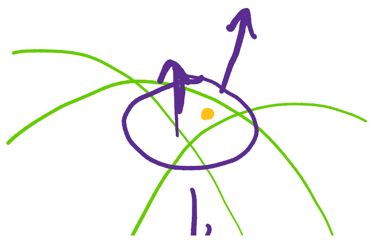
$$\boxed{x^2+y^2+\varphi(x,y)^2 = 3xy\varphi(x,y) \qquad \varphi(1,1)=1}$$



$$f(x,y,\varphi(x,y)) = 0$$

$$\frac{\partial}{\partial x}\quad 0 = 1\cdot\frac{\partial f}{\partial x}(x,y,\varphi(x,y)) + 0\cdot\frac{\partial f}{\partial y}(x,y,\varphi(x,y)) + \frac{\partial \varphi}{\partial x}(x,y)\cdot\frac{\partial f}{\partial z}(x,y,\varphi(x,y))$$

$$\boxed{\frac{\partial\varphi}{\partial x}(x,y) = -\frac{\frac{\partial f}{\partial x}(x,y,\varphi(x,y))}{\frac{\partial f}{\partial z}(x,y,\varphi(x,y))}}$$



$$f(x,y,z) = x^2+y^2+z^2-3xyz$$
$$1 \quad 1 \quad 1 - 3 = 0 \quad \checkmark$$
$$f(1,1,1) = 0$$

$$\frac{\partial f}{\partial z} = 2z-3xy \qquad \frac{\partial f}{\partial z}(1,1,1) = 2-3 = \underline{-1} \ (\neq 0)$$

$$\frac{\partial f}{\partial x} = 2x-3yz \qquad \frac{\partial f}{\partial y} = 2y-3xz$$

$$\frac{\partial f}{\partial x}(1,1,1) = \frac{\partial f}{\partial y}(1,1,1) = \frac{\partial f}{\partial z}(1,1,1) = \underline{\underline{-1}}$$

$$\frac{\partial \varphi}{\partial x}(1,1) = \frac{\partial \varphi}{\partial y}(1,1) = -1 \quad \left(-\frac{-1}{-1}\right)$$

- $\nabla\varphi(1,1) = \underline{\underline{(-1,-1)}}$

$(1,2,0,0)$

(8) $\left|\begin{array}{l} xe^{u+v}+2uv-1=0 \\ ye^{u-v}-\frac{u}{1+v}-2x=0 \end{array}\right| \begin{array}{l} \to 1\cdot 1+0-1=0 \ \checkmark \\ \to 2\cdot 1-0-2=0 \ \checkmark \end{array}$

$e^{u+v} = 1$, $e^{u-v} = 1$

- $F(x,y,\underline{\underline{u}},\underline{\underline{v}}) = \left(xe^{u+v}+2uv-1,\ ye^{u-v}-\frac{u}{1+v}-2x\right)$

$F:\mathbb{R}^4 \to \mathbb{R}^2 \qquad \varphi:\mathbb{R}^2\to\mathbb{R}^2 \qquad \varphi = (u,v)$

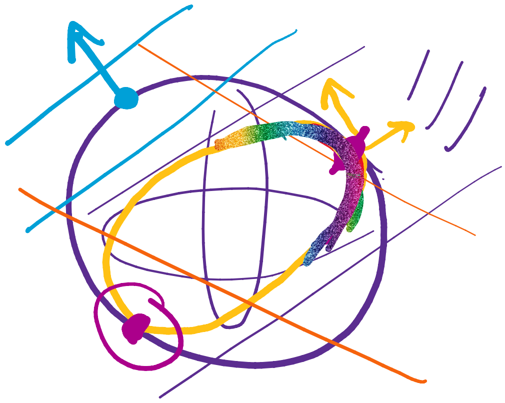
$$J_F(x,y,u,v) = \left(\begin{array}{cc|cc} e^{u+v}, & 0, & xe^{u+v}+2v, & xe^{u+v}+2u \\ -2 & e^{u-v} & ye^{u-v}-\frac{1}{\ldots} & ye^{u-v}+\frac{u}{\ldots} \end{array}\right.$$

$$V_F(x,y,u,v) = \left(-2,\ e^{u-v}\ \Big|\ ye^{u-v} - \frac{1}{1+v},\ -ye^{u-v} + \frac{u}{(1+v)^2} \right)$$

$$\boxed{J_x} \qquad \boxed{J_u}$$

- $J_x(1,2,0,0) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$
- $J_u(1,2,0,0) = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$ $\quad \det\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} = -2-1 = -3 \neq 0$

$x^2+y^2+z^2=1$



$x+y+z=0$

$$F: \mathbb{R}^{d+m} \to \mathbb{R}^m$$

$$\varphi = (u,v)$$

$$F(x,y,\underbrace{u(x,y),v(x,y)}) = 0 \in \mathbb{R}^2$$

$$F(x,y,\varphi(x,y)) = 0 \in \mathbb{R}^2$$

$$J_\varphi(1,2) = -\left[J_u(1,2,0,0)\right]^{-1} \cdot J_x(1,2,0,0)$$

$$\left(J_u(1,2,0,0)\right)^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}^{-1} = \frac{1}{-3}\begin{pmatrix} -2 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{pmatrix}$$

$$J_x(1,2,0,0) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$J_\varphi(1,2) = -\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = -\begin{pmatrix} 0 & \frac{1}{3} \\ 1 & -\frac{1}{3} \end{pmatrix} = \boxed{\begin{pmatrix} 0 & -\frac{1}{3} \\ -1 & \frac{1}{3} \end{pmatrix}}$$

$\varphi = (u,v)$

$$\frac{\partial u}{\partial x}(1,2) = 0 \qquad \frac{\partial u}{\partial y}(1,2) = -\frac{1}{3}$$

$$\frac{\partial v}{\partial x}(1,2) = -1 \qquad \frac{\partial v}{\partial y}(1,2) = \frac{1}{3}$$

(10) $\boxed{x = e^u + u\sin v \qquad y = e^u - u\cos v}$

$F : (u,v) \to (x,y)$ $\quad F^{-1} : \mathbb{R}^2 \to \mathbb{R}^2$

$(x,y) = F(u,v)$

$$\left(e^u + u\sin v - x,\ e^u - u\cos v - y\right)$$

$$J_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \boxed{J_{F^{-1}}} = \cancel{-}(J_u)^{-1}\cancel{\left(-\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)}$$

$$= (J_u)^{-1} = \boxed{(J_F)^{-1}}$$

(3), (7), (9) $\to$ $\varphi'(0) = 2$, $\varphi''(0) = -14$