Stochastic processes 2 – first test

1. Let $\{W_t, t \geq 0\}$ be a Wiener process (recall that it has independent increments). The process $\{Y_t, t \geq 0\}$ is defined as $Y_t = W_{t+2} - W_t, t \geq 0$.

   (a) Determine the mean value function and the autocovariance function of $\{Y_t, t \geq 0\}$. [2 pts]

   (b) Decide whether the process $\{Y_t, t \geq 0\}$ is weakly stationary. [1 pt]

2. A real-valued weakly stationary random sequence $\{U_t, t \in \mathbb{Z}\}$ has the following autocovariance function: $R_U(k) = (1/6)^k, k = 0, 1, 2, \dots$ Determine the values $R_U(-1), R_U(-2), \dots$ and decide whether the spectral density of the sequence exists. If it does, calculate it. [4 pts]

3. A real-valued random process $\{D_t, t \geq 0\}$ is centered and has the following autocovariance function, $s, t \geq 0$: $R_D(s,t) = 2s - t$ for $s \leq t < 2s$, $R_D(s,t) = 2t - s$ for $t < s < 2t$, $R_D(s,t) = 0$ otherwise. Is the process $\{D_t, t \geq 0\}$ L_2-continuous and L_2-differentiable? Is it L_2-integrable on a bounded interval $[a, b] \subset [0, \infty)$? [4 pts]

4. Consider a weakly stationary random sequence $\{X_t, t \in \mathbb{Z}\}$ with the spectral density $f_X(\lambda) = 3 \cdot \mathbb{I}_{(-\pi/3, \pi/3)}(\lambda), \lambda \in [-\pi, \pi]$. Determine the spectral distribution function and the autocovariance function of the sequence $\{X_t, t \in \mathbb{Z}\}$. [3 pts]

For successfully passing this test obtaining ***at least 10 points*** *out of 14 is required.*

Stochastic processes 2 – second test

1. Consider an AR(2) process $\{X_t, t \in \mathbb{Z}\}$ given by the formula

   $$X_t - \frac{11}{12} X_{t-1} + \frac{1}{6} X_{t-2} = Y_t, \quad t \in \mathbb{Z},$$

   where $\{Y_t, t \in \mathbb{Z}\}$ is a centered real-valued white noise with positive finite variance σ^2.

   (a) Determine the autocovariance function of the process $\{X_t, t \in \mathbb{Z}\}$ using the Yule-Walker equations. You do not have to calculate the precise value of $R(0)$, just express it using fractions. [6 pts]

   (b) Suppose that we know the whole history of the process up to time t. Find the prediction of X_{t+1} and X_{t+2} and determine the prediction error. [3 pts]

2. Consider an ARMA(1,1) process $\{X_t, t \in \mathbb{Z}\}$ given by the formula

   $$X_t - \frac{1}{5} X_{t-1} = Y_t + \frac{3}{4} Y_{t-1}, \quad t \in \mathbb{Z},$$

   where $\{Y_t, t \in \mathbb{Z}\}$ is a centered real-valued white noise with positive finite variance σ^2. Discuss the causality of the process $\{X_t, t \in \mathbb{Z}\}$ and find the corresponding MA(∞) representation. Determine the coefficients of the corresponding linear filter and calculate its transfer function. [3 pts]

3. Show that a sequence $\{Z_t, t \in \mathbb{Z}\}$ following a MA(3) model is mean square ergodic and write down what does that mean for the sequence $\frac{1}{n} \sum_{i=1}^{n} Z_n, n \in \mathbb{N}$. [2 pts]

For successfully passing this test obtaining ***at least 10 points*** *out of 14 is required.*