Functional Change-Point Analysis with Increasing Number of Projections

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Abstract

Functional principal components (FPC's) provide the most important and most extensively used tool for dimension reduction and inference for functional data (see, e.g., Horváth and Kokoska (2012) for a recent overview). The selection of the number, d, of the FPC's to be used in a specific procedure has attracted a fair amount of attention, and a number of reasonably effective approaches exist. Intuitively, they assume that the functional data can be sufficiently well approximated by a projection onto a finite-dimensional subspace, and the error resulting from such an approximation does not impact the conclusions. This has been shown to be a very effective approach, but it is desirable to understand the behavior of many inferential procedures by considering the projections on subspaces spanned by an increasing number of the FPC's. Such an approach reflects more fully the infinite-dimensional nature of functional data, and allows to derive procedures which are fairly insensitive to the selection of d. This is accomplished by considering limits as $d \to \infty$ with the sample size.

In this talk we present some results from Fremdt et al. (2012), wherein a specific framework has been proposed in which we let $d = d(N) \to \infty$ by deriving a normal approximation for the (two-parameter) partial sum process

$$\sum_{j=1}^{\lfloor du \rfloor} \sum_{i=1}^{\lfloor Nx \rfloor} \xi_{i,j}, \quad 0 \le u \le 1, \quad 0 \le x \le 1,$$

where N is the sample size and $\xi_{i,j}$ is the score of the i-th function with respect to the j-th FPC. Our approximation can be used to derive statistics that use segments of observations and segments of the FPC's. We apply our general results to derive inferential procedures for the mean function, in particular, a change-point test in an a-posteriori setting is studied. In addition to the asymptotic theory, the test is also assessed through a small simulation study and a real data example.

The talk is based on joint work with Stefan Fremdt (Cologne), Lajos Horváth (Salt Lake City), and Piotr Kokoszka (Fort Collins).