1BP453 - Computational Finance

Seminar Paper - afternoon class

Send your solutions by email to jcerny@karlin.mff.cuni.cz no later than November 20th, 2016 at 11:59 pm. With each Problem, please, send me commented Matlab (or other used programming language) code, results and your explanation in PDF or Word format.

Problem 1 (30 points)**:** Let us have a system of linear equations

$$\begin{array}{rcrcrcr} 10x_1 & - & 2x_2 & + & 3x_3 & = & 3, \\ x_1 & + & 4x_2 & + & x_3 & = & 1, \\ -2x_1 & - & x_2 & + & 10x_3 & = & -3. \end{array}$$

Prove that Jacobi and Gauss–Seidel methods converge, evaluate first four iterations, and estimate the error of the last iteration using both methods (take the initial iteration $\bar{x}^{(0)} = (0,0,0)'$). Compare these error estimations with the exact error calculated from the solution of the system and discuss the results.

Problem 2 (15 points)**:** Find a root of the equation

$$x^3 + 3x^2 - 12x + 5 = 0$$

in the interval $(0;1)$ using secant and Newton's method for $n = 3$. Sketch the graphs of all curves (including cubic function) into one readable chart. Calculate the estimation of error and the exact error for both methods. Discuss the results.

Problem 3 (15 points)**:** For the nonlinear system

$$\begin{array}{rcl} x^2 + 2y^2 & = & 2 \\ y & = & \mathrm{e}^x - 1 \end{array}$$

evaluate four iterations using Newton's method with the initial iteration $\bar{x}^{(0)} = (1,2)'$.

Problem 4 (10 points)**:** Evaluate the integral $I_{\text{indef}} = \int \exp\{2x - 1\} dx$ using the symbolic toolbox in Matlab and make the plot of the integral on the interval $(1, 2)$. Evaluate definite integral

$$I_{(1,2)} = \int_1^2 \exp\{2x - 1\} dx$$

using trapezoid method for $n = 4$, estimate the error, and compare it with the exact error. Discuss the results.

Problem 5 (30 points)**:** Solve the differential equation

$$y' - \frac{1}{2}y - x - 1 = 0$$

using Euler, Modified Euler, and Runge-Kutta method of the 4th order (with the same setup as it was used in the lecture) with the initial condition $y(0) = 1$ on $\langle 0; 0,4 \rangle$, $h = 0,1$. The exact solution of this ODE is function $\phi^*(x) = 7e^{x/2} - 6 - 2x$. Calculate the exact error and sketch the graphs of all functions (obtained results and exact solution) at particular points into one readable chart. Discuss the results.