1BP453 - Computational Finance

SEMINAR PAPER - morning class

Send your solutions by email to jcerny@karlin.mff.cuni.cz no later than November 20th, 2016 at 11:59 pm. With each Problem, please, send me commented Matlab (or other used programming language) code, results and your explanation in PDF or Word format.

Problem 1 (25 points)**:** Let us have a system of linear equations

$$\begin{aligned} 4x_1 &- 2x_2 + x_3 = -3, \\ x_1 &- 8x_2 + 2x_3 = 2, \\ x_1 &\quad\quad + 4x_3 = 12. \end{aligned}$$

Prove that Jacobi and Gauss–Seidel methods converge, evaluate first three iterations, and estimate the error of the last iteration using both methods (take the initial iteration $\bar{x}^{(0)} = (0, 0, 0)'$). Compare these error estimations with the exact error calculated from the solution of the system and discuss the results.

Problem 2 (10 points)**:** Find a root of the equation

$$x^3 - 2x^2 - 3x + 1 = 0$$

in the interval $(2; 4)$ using secant method for $n = 4$. Sketch the graphs of all curves (secants and cubic function) into one chart.

Problem 3 (15 points)**:** For the nonlinear system

$$\begin{aligned} x^2 + y^2 &= 4 \\ y &= \mathrm{e}^{2x} - 1 \end{aligned}$$

evaluate two iterations using Newton's method with the initial iteration $\bar{x}^{(0)} = (1, 2)'$.

Problem 4 (10 points)**:** Evaluate the integral $I_{\text{indef}} = \int \exp\{2x + 1\}dx$ using the symbolic toolbox in Matlab and make the plot of the integral on the interval $(0, 1)$. Evaluate definite integral

$$I_{(0,1)} = \int_0^1 \exp\{2x + 1\}dx$$

using Simpson's method for $n = 4$, estimate the error, and compare it with the exact error. Discuss the results.

Problem 5 (25 points)**:** Solve the differential equation

$$y' = x + y + 2$$

using Euler method with the initial condition $y(0) = 1$ on $\langle 0; 0, 4\rangle$, $h = 0, 1$. The exact solution of this ODE is function $\phi^*(x) = 4e^x - 3 - x$. Calculate the exact error and sketch the graphs of both functions (obtained result and exact solution) at particular points. Discuss the results.

Problem 6 (15 points)**:** Evaluate the price of European put option (at time 0) if the initial asset price is $S_0 = 100$, option maturity $T = \frac{5}{12}$, strike price is $K = 100$, risk-free interest rate $r = 0, 01$, and volatility $\sigma = 0, 4$. Use Black-Scholes PDE and solve it using finite difference method where $S_{max} = 200$, $\delta t = \frac{5}{1200}$, $\delta S = 8$. Verify the solutions using the in-built Matlab function *blsprice* and explain the obtained results.