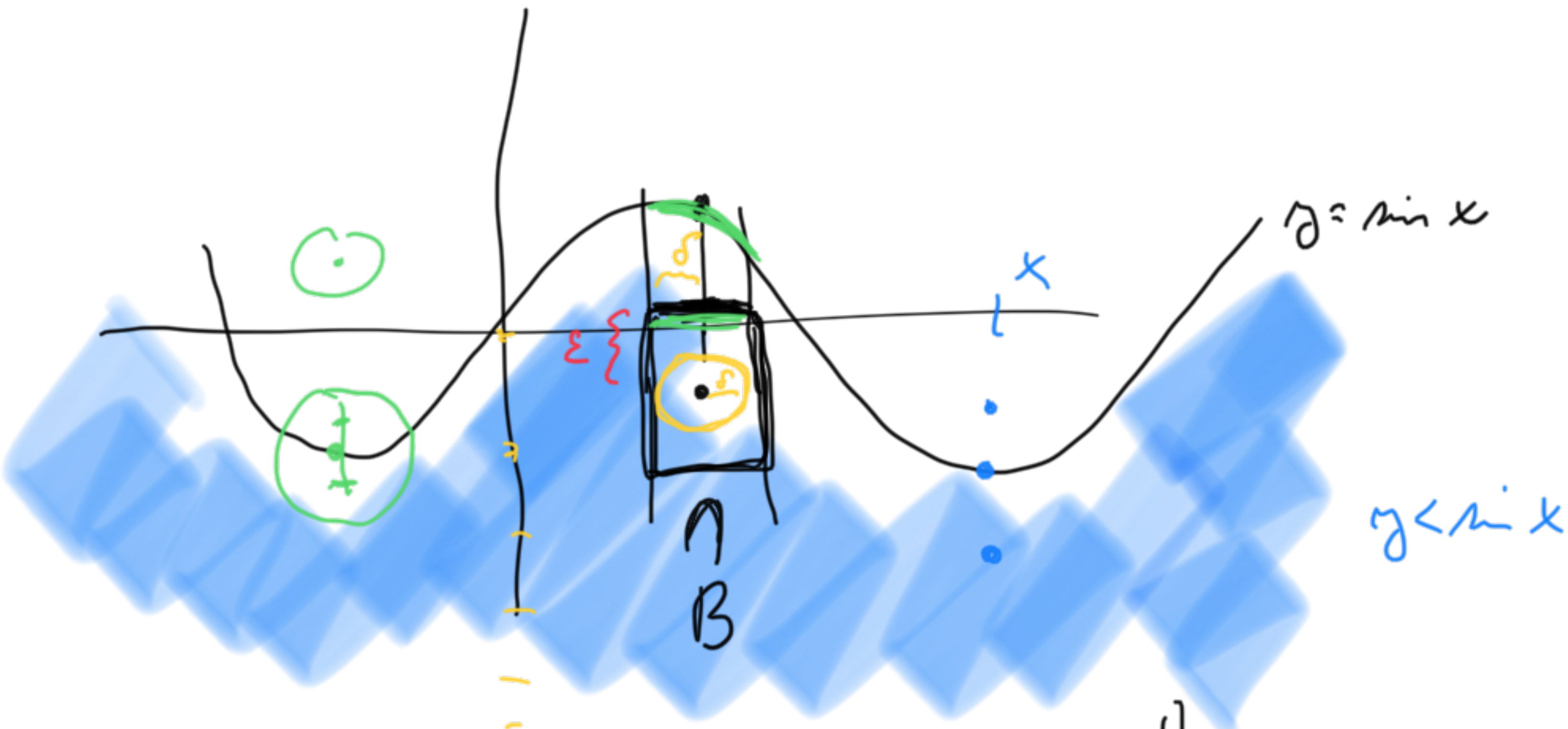
$$B = \{[x,y] \in \mathbb{R}^2;\ y < \sin x\} = \{[x,y]\in\mathbb{R}^2;\ \underbrace{\sin x - y}_{f(x,y)} > 0\} = \{[x,y]\in\mathbb{R}^2;\ f(x,y) > 0\}$$

otevřená dle V12

$f(x,y) = \sin x - y$ ... spojitá fce na $\mathbb{R}^2$



$H(B) = \{[x,y];\ y = \sin x\}$

$\overline{B} = \{[x,y];\ y \le \sin x\}$

$[a,b] \in B$

$b < \sin a$

$\exists \varepsilon > 0:\ b + \varepsilon < \sin a$

sin je spojitá v a ⇒

$\exists \delta > 0:\ \forall x \in (a-\delta, a+\delta):\ \sin x > b + \varepsilon$

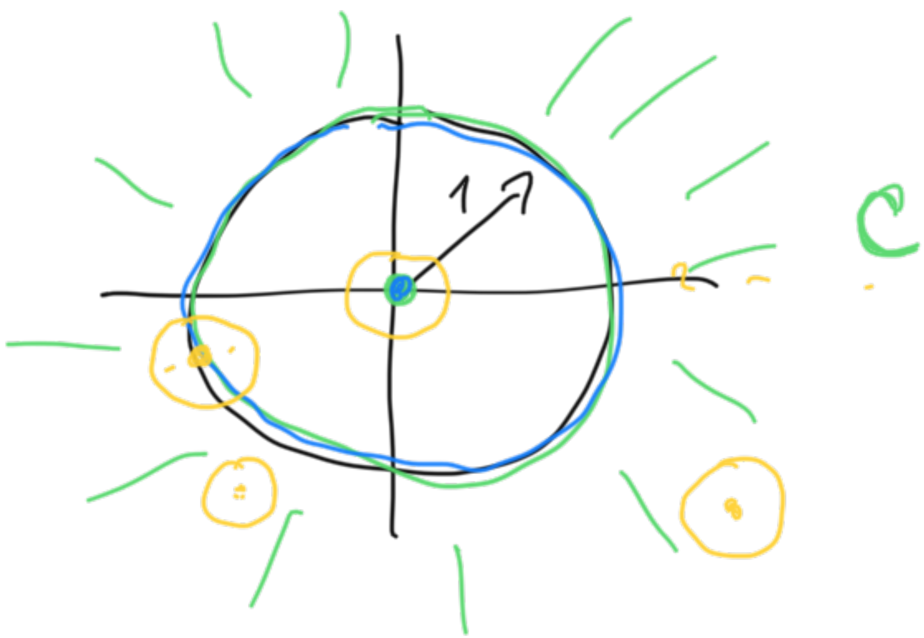
$r = \min\{\delta, \varepsilon\}$, pak

$B([a,b], r) \subset$ „obdélník"

$C = \{ [x,y] \in \mathbb{R}^2 ;\ \underbrace{\overbrace{(x^2+y^2)}^{\geq 0}\overbrace{(1-x^2-y^2)}^{\leq 0}}_{f} \leq 0 \}$

V12 $\Rightarrow$ uzavřená

$\{ 1-x^2-y^2 \leq 0 \} = \{ x^2+y^2 \geq 1 \} = \{ \rho([x,y],o) \geq 1 \}$



$\bar{C} = C$

$H(C) = \{ [x,y] ;\ x^2+y^2 = 1 \} \cup \{ [0,0] \}$

$\operatorname{Int} C = C \setminus H(C) = \{ [x,y] ;\ x^2+y^2 > 1 \}$

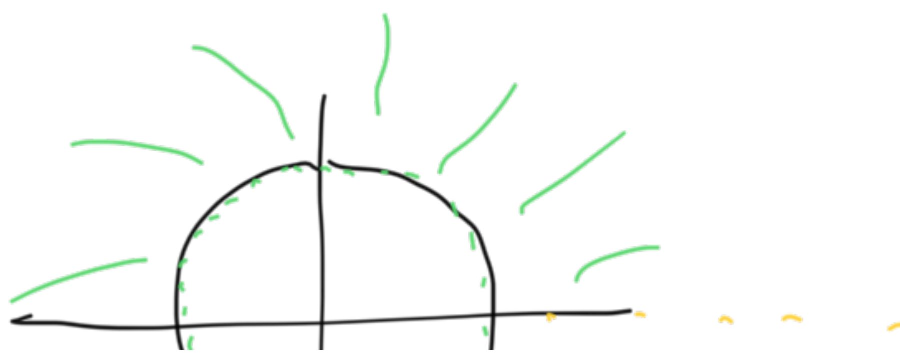
$H(C) = \{ (x^2+y^2)(1-x^2-y^2) = 0 \}$

POZOR!

$D = \{ [x,y] \in \mathbb{R}^2 ;\ (x^2+y^2)(1-x^2-y^2) < 0 \}$ $= \{ x^2+y^2 > 1 \}$

otevřená (V12)

$D = \operatorname{Int} C$



$H(D) = \{x^2+y^2=1\} \neq \{(x^2+y^2)(1-x^2-y^2)=0\}$

$\bar{D} = \{x^2+y^2 \geq 1\} \neq \{(x^2+y^2)(1-x^2-y^2) \leq 0\}$

$E = \{[x,y] \in \mathbb{R}^2;\ 5x^2+2y^2+2xy \leq 5\}$

$f(x,y)$ spojitá

je uzavřená (V12) $\Rightarrow H(E) \subset E$

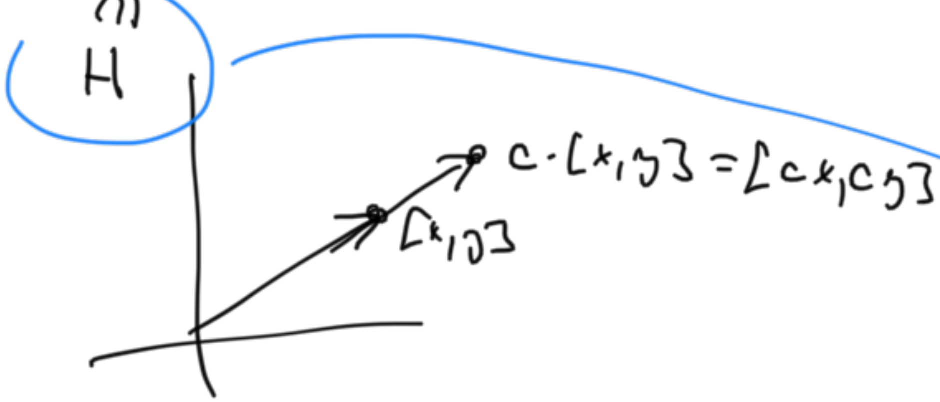
$\{5x^2+2y^2+2xy<5\} \subset E$ otevřená (V12)

$H(E) \subset \{5x^2+2y^2+2xy=5\}$

hyp: $H(E) = \{5x^2+2y^2+2xy=5\}$ H

$[x,y] \rightsquigarrow c\cdot[x,y]$: $f(cx,cy) = 5c^2x^2+2c^2y^2+2cxcy = c^2(5x^2+2y^2+2xy) = 5c^2$

$=5$

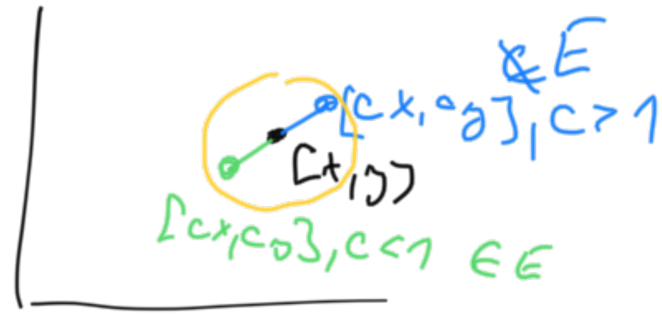


$c>1$: $f(cx,cy)>5 \Rightarrow [cx,cy] \notin E$

$0<c<1$: $f(cx,cy)<5 \Rightarrow [cx,cy] \in E$

$c < 1$: $q(cx, cy) < 5 \Rightarrow [cx, cy] \in E$

$\Rightarrow [x, y] \in H(E)$



omezenost:

$$5x^2 + 2y^2 + 2xy = 4x^2 + y^2 + (x+y)^2 \geq x^2 + y^2$$

$$[x, y] \in E \Rightarrow \quad \leq 5 \quad \Rightarrow x^2 + y^2 \leq 5 \Rightarrow [x, y] \in B(0, \sqrt{5})$$

$$F = \{[x, y, z];\ 2 \leq xyz < 4\}$$

$$\overline{F} = \{2 \leq xyz \leq 4\} = \{xyz \geq 2 \ \&\ xyz \leq 4\} = \{xyz \geq 2\} \cap \{xyz \leq 4\}$$

spojitá

uzavřená

V12 — uz. — uz.

průnik uzavřených

$$\operatorname{Int} F = \{2 < xyz < 4\}$$

otevřená (průnik 2 otevřených)

$\{2 \leq xy < 4\}$

TIP: $H(F) = \{xyr=2\} \cup \{xyr=4\}$

F není omezená

$r = 1$

$\{[x,y,1];\ 2 \leq xy\cdot 1 < 4\}$

$\vec{a}_n = [\frac{1}{n}, 2n, 1] \in F$

$\rho(\vec{a}_n, o) \geq 2n \to +\infty$