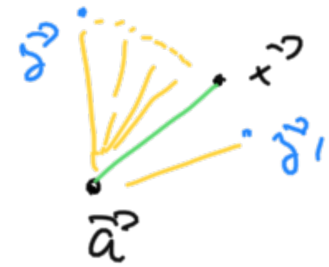
Zvolme pevně $\vec{a} \in \mathbb{R}^n$.



Fce $\vec{x} \mapsto \rho(\vec{x}, \vec{a})$ je spojitá na $\mathbb{R}^n$.

$$\rho(\vec{x}, \vec{a}) = \sqrt{\sum_{i=1}^{n} (x_i - a_i)^2}$$ ... spojitá (složení spojitých)

aritmetika spoj. fcí

$\vec{x} \mapsto x_1 = \pi^1(\vec{x})$ $\vec{x} \mapsto x_2 = \pi^2(\vec{x})$ $\vdots$ spojité	$\vec{x} \mapsto x_1 - a_1$ $\vec{x} \mapsto x_2 - a_2$ spojité $\vdots$	$\vec{x} \mapsto (x_i - a_i)^2$ spojité	$\vec{x} \mapsto \sum_{i=1}^{n} (x_i - a_i)^2$ spojitá

jinak:

$\vec{x}, \vec{y} \in \mathbb{R}^n$:

ve skutečnosti: $\rho(\vec{x}, \vec{a}) \le \rho(\vec{x}, \vec{y}) + \rho(\vec{y}, \vec{a})$, tj.

$\rho(\vec{y}, \vec{a}) \le \rho(\vec{y}, \vec{x}) + \rho(\vec{x}, \vec{a})$

$$\left.\begin{aligned} &\rho(\vec{x}, \vec{a}) - \rho(\vec{y}, \vec{a}) \le \rho(\vec{x}, \vec{y}) \quad \& \\ &\rho(\vec{y}, \vec{a}) - \rho(\vec{x}, \vec{a}) \le \rho(\vec{x}, \vec{y}) \end{aligned}\right\}$$

$$\left| \rho(\vec{x}, \vec{a}) - \rho(\vec{y}, \vec{a}) \right| \le \rho(\vec{x}, \vec{y})$$

$|a+b| \le |a| + |b|$

$\big| |x-a| - |y-a| \big| \le |x-y|$

Z definice: zvolme $\varepsilon > 0$, položme $\delta = \varepsilon$.

Pak $\forall \vec{y} \in B(\vec{x},\delta)$:

$$\big|\underbrace{\rho(\vec{x},\vec{a})}_{f(\vec{x})}-\underbrace{\rho(\vec{y},\vec{a})}_{f(\vec{y})}\big| \le \rho(\vec{x},\vec{y}) < \delta = \varepsilon$$

$\vec{y} \mapsto \rho(\vec{y},\vec{a})$, spojitost v bodě $\vec{x}$

$f(\vec{y}) = \rho(\vec{y},\vec{a})$

$$f(x,y) = \frac{\cos(xy)}{\sqrt{x^2+y^2}}$$ ..... spoj. fce na $\mathcal{D}_f = \mathbb{R}^2 \setminus \{[0,0]\}$

$[x,y] \mapsto x = \pi^1(x,y)$

$[x,y] \mapsto y = \pi^2(x,y)$

spojité na $\mathbb{R}^n$

aritmetika

$[x,y] \mapsto x\cdot y$

$[x,y] \mapsto x^2+y^2$

spojité na $\mathbb{R}^2$

skládání spoj. fcí

$[x,y] \mapsto \cos(x\cdot y)$

$[x,y] \mapsto \sqrt{x^2+y^2}$

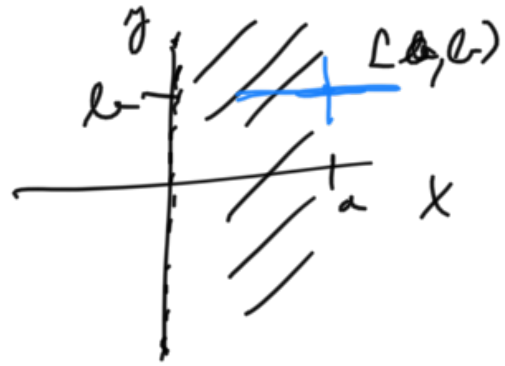
spojité na $\mathbb{R}^2$

podíl spoj. fcí

$[x,y] \mapsto \dfrac{\cos(xy)}{\sqrt{x^2+y^2}}$ spojitá všude tam, kde $\sqrt{x^2+y^2} \neq 0$, tj.: $[x,y] \neq [0,0]$

$f(x,y) = x^y = \exp(y\cdot\log x)$

$\mathcal{D}_f = \{[x,y];\ x>0\} = (0,+\infty)\times\mathbb{R}$ ... otevřená



$[a,b] \in \mathcal{D}_f$

$\dfrac{\partial f}{\partial x}(a,b) = g'(a) = b\cdot a^{b-1}$ | $\dfrac{\partial f}{\partial x}(x,y) = y\cdot x^{y-1}$, $x>0$

$$g(x) = f(x,b) = x^{b}$$

$$g'(x) = b\,x^{b-1}$$

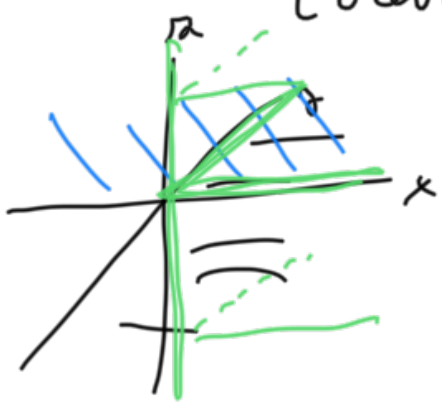
$$\frac{\partial f}{\partial y}(x,y) = \exp(y \cdot \log x) \cdot \underbrace{\frac{\partial h}{\partial y}(x,y)}_{\log x} = x^{y} \cdot \log x\,, \quad \begin{matrix} x>0 \\ y\in\mathbb{R} \end{matrix}$$

$$h(x,y) = y \cdot \log x$$

$$f(x,y,R) = x^{(y^R)}$$

$$D_f = \{ (x,y,R) \,;\, x>0,\ y>0 \} = (0,+\infty) \times (0,+\infty) \times \mathbb{R} = (0,+\infty)^2 \times \mathbb{R}$$

(otevřená množina)



$$\frac{\partial f}{\partial x}(x,y,R) = \frac{\partial (x^{\text{čísl.}})}{\partial x} = (y^R) \cdot x^{(y^R - 1)} \quad , \begin{matrix} x>0 \\ y>0 \\ R\in\mathbb{R} \end{matrix}$$

$$f(x,y,R) = \exp(y^R \cdot \log x) = \exp(\exp(R \cdot \log y) \cdot \log x)$$

$$\frac{\partial f}{\partial y}(x,y,z) = \exp(y^R \cdot \log x) \cdot \log x \cdot R \cdot y^{R-1} = x^{(y^R)} \ldots {}^{R-1}$$

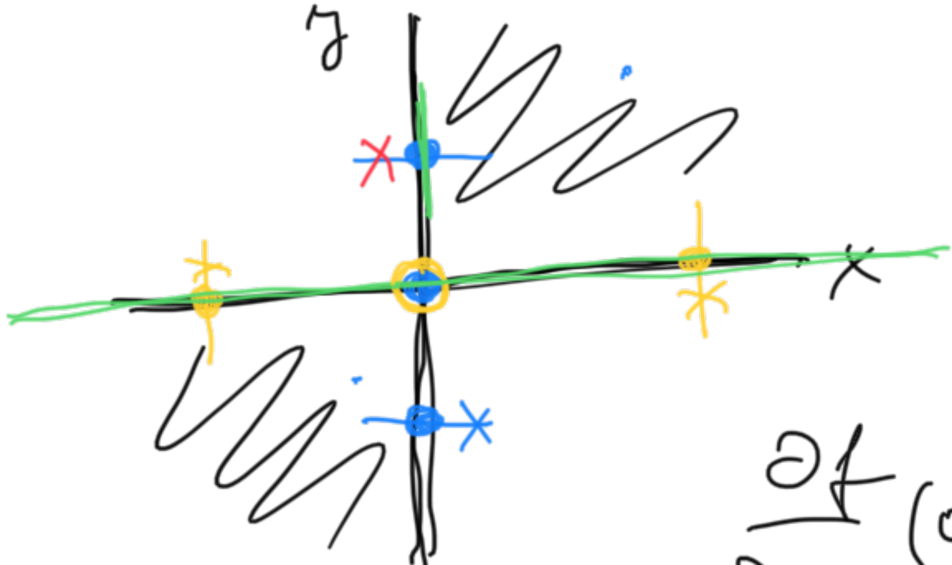
$$\frac{\partial f}{\partial y}(x,y,R) = \underbrace{\exp(y^R \cdot \log x)}_{x^{(y^R)}} \cdot \log x \cdot R \cdot y^{R-1} = x^{(y^R)} \cdot y^{R-1} \cdot R \cdot \log x, \quad \begin{matrix} x>0 \\ y>0 \\ R\in\mathbb{R} \end{matrix}$$

$$\frac{\partial f}{\partial R}(x,y,R) = \exp(y^R \cdot \log x) \cdot \log x \cdot y^R \cdot \log y =$$

$$= x^{(y^R)} \cdot y^R \cdot \log y \cdot \log x, \quad \begin{matrix} x>0 \\ y>0 \\ R\in\mathbb{R} \end{matrix}$$

$$f(x,y) = \sqrt{x\cdot y}$$

$$D_f = \{[x,y];\ x\cdot y \geq 0\} = \{[x,y];\ x\geq 0 \,\&\, y \geq 0\} \cup \{[x,y];\ x\leq 0 \,\&\, y\leq 0\}$$ (uzavřená mn.)



$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2}\frac{1}{\sqrt{x\cdot y}}\cdot y \;\cancel{= \frac{1}{2}\cdot\sqrt{\frac{y}{x}}}, \quad x>0 \,\&\, y>0 \ \vee\ x<0 \,\&\, y<0$$

$$\frac{\partial f}{\partial x}(0,y)$$ neexistuje pro $y \neq 0$ (kvůli def. oboru)

$$\frac{\partial f}{\partial x}(x,0) = 0, \quad x\in\mathbb{R}$$

$$g(x) = f(x,0) = 0$$

symetrie:

$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{2}\frac{1}{\sqrt{xy}}\cdot x, \quad x>0 \,\&\, y>0 \ \vee\ x<0 \,\&\, y<0$$

$$\cancel{= \frac{1}{2}\sqrt{\frac{x}{y}}}$$

$$\frac{\partial f}{\partial y}(x,0) \text{ neexistuje} \quad , \; x \neq 0$$

$$\frac{\partial f}{\partial y}(0,y) = 0 \quad , \; y \in \mathbb{R}$$