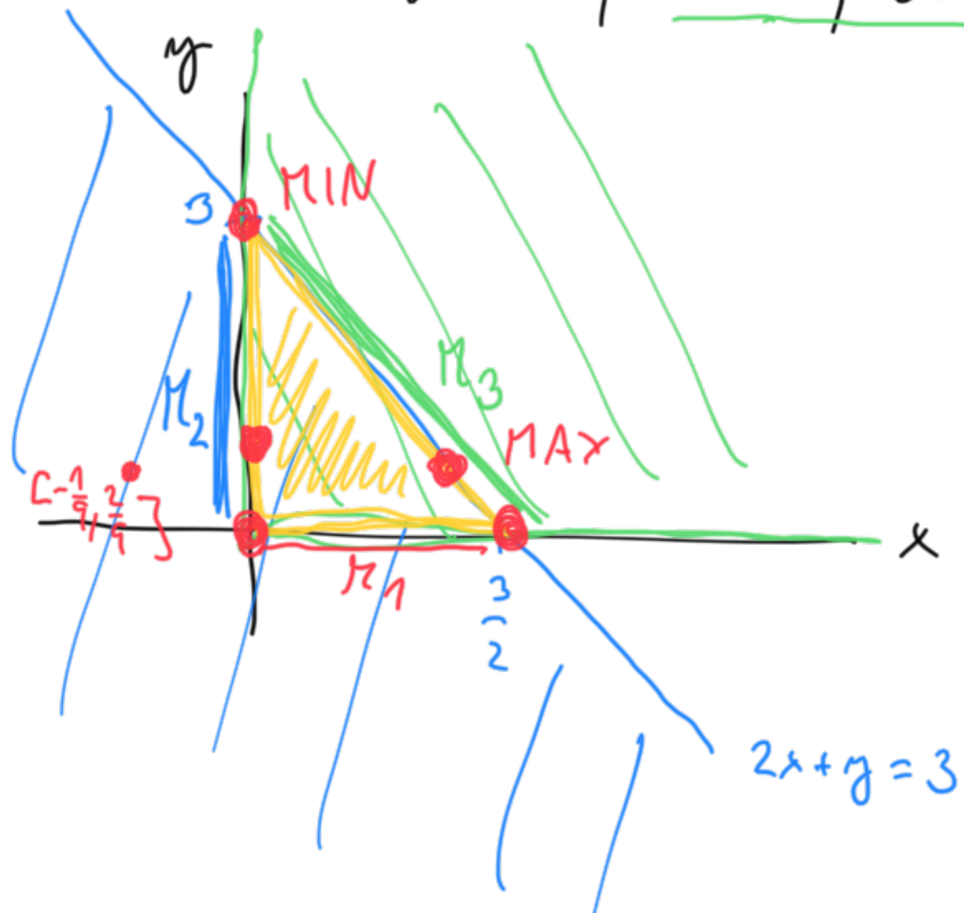
$$f(x,y) = x^2 - 2y^2 + xy + y$$

najděte extrémy f na M
(nebo aspoň sup / inf)

$$M = \{ [x,y] \in \mathbb{R}^2 : x \geq 0, y \geq 0, 2x+y \leq 3 \}$$



M je omezená
M je uzavřená (průnik 3 uzavřených) } kompaktní

f je spojitá $\Rightarrow$ nabývá na M max. i min.

$$M = \operatorname{Int} M \cup \underbrace{M_1 \cup M_2 \cup M_3}_{\partial(M)}$$

$$\operatorname{Int} M = \{ x > 0, y > 0, 2x + y < 3 \}$$
$$M_1 = \{ [x, 0], x \in \langle 0, \tfrac{3}{2} \rangle \}$$
$$M_2 = \{ [0, y], y \in \langle 0, 3 \rangle \}$$
$$M_3 = \{ 2x + y = 3, x \geq 0, y \geq 0 \}$$

na Int M: $f \in C^1(\operatorname{Int} M)$, hledáme kritické body f na Int M

$$\nabla f(x,y) = [2x + y, -4y + x + 1]$$
$$= o \Leftrightarrow \begin{array}{l} 2x + y = 0 \\ x - 4y = -1 \end{array} \quad \& \quad y = -2x$$

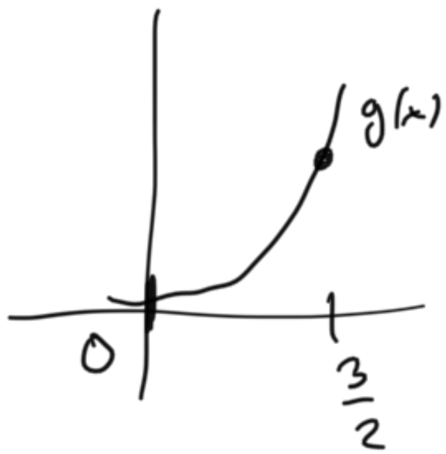
$$x + 8x = -1$$

$4x = -1$

$x = -\frac{1}{9}$, $y = \frac{2}{9}$ $\notin \operatorname{Int} M$

Tedy na Int M není žádný bod „podezřelý z extrému".

na M_1: $g(x) = f(x,0) = x^2$, $x \in \langle 0, \frac{3}{2} \rangle$



na M_1 má f min v $[0,0]$, $f(0,0) = 0$

max v $[\frac{3}{2}, 0]$, $f(\frac{3}{2}, 0) = \frac{9}{4}$

na M_2: $g(y) = f(0,y) = -2y^2 + y$, $y \in \langle 0, 3 \rangle$

$g'(y) = -4y + 1$, $y \in (0,3)$

$= 0 \iff y = \frac{1}{4} \in (0,3)$

Body podezřelé z extrému:

$[0,0]$

$[0,3]$

$[0,\frac{1}{4}]$

na M_3: $M_3 = \{ y = 3 - 2x, \ x \in \langle 0, 3 \rangle \}$

$h_3 = \{ y = 3-2x,\ x \in \langle 0, \frac{3}{2} \rangle \}$

$$g(x) = f(x, 3-2x) = x^2 - 2(3-2x)^2 + x(3-2x) + 3-2x =$$
$$= x^2 - 2(9 - 12x + 4x^2) + 3x - 2x^2 + 3 - 2x =$$
$$= x^2 - 18 + 24x - 8x^2 + 3x - 2x^2 + 3 - 2x =$$
$$= -9x^2 + 25x - 15, \quad x \in \langle 0, \tfrac{3}{2} \rangle$$

$$g'(x) = -18x + 25, \quad x \in (0, \tfrac{3}{2})$$
$$= 0 \iff x = \frac{25}{18} \in (0, \tfrac{3}{2})$$

Body podezřelé z extrému:

$[0, 3]$

$[\frac{3}{2}, 0]$

$[\frac{25}{18}, \frac{4}{18}] = [\frac{25}{18}, \frac{2}{9}]$

Dosadíme:

$f(0,0) = 0$

$f(\frac{3}{2}, 0) = \frac{9}{4}$

$f(0,3) = -18 + 3 = -15$ min

$f(0, \frac{1}{4}) = -\frac{2}{16} + \frac{1}{4} = \frac{1}{8}$

$f(\frac{25}{18}, \frac{2}{9}) = (\frac{25}{18})^2 - 2 \cdot (\frac{2}{9})^2 + \frac{25}{18} \cdot \frac{2}{9} + \frac{2}{9} = \frac{85}{36}$ max

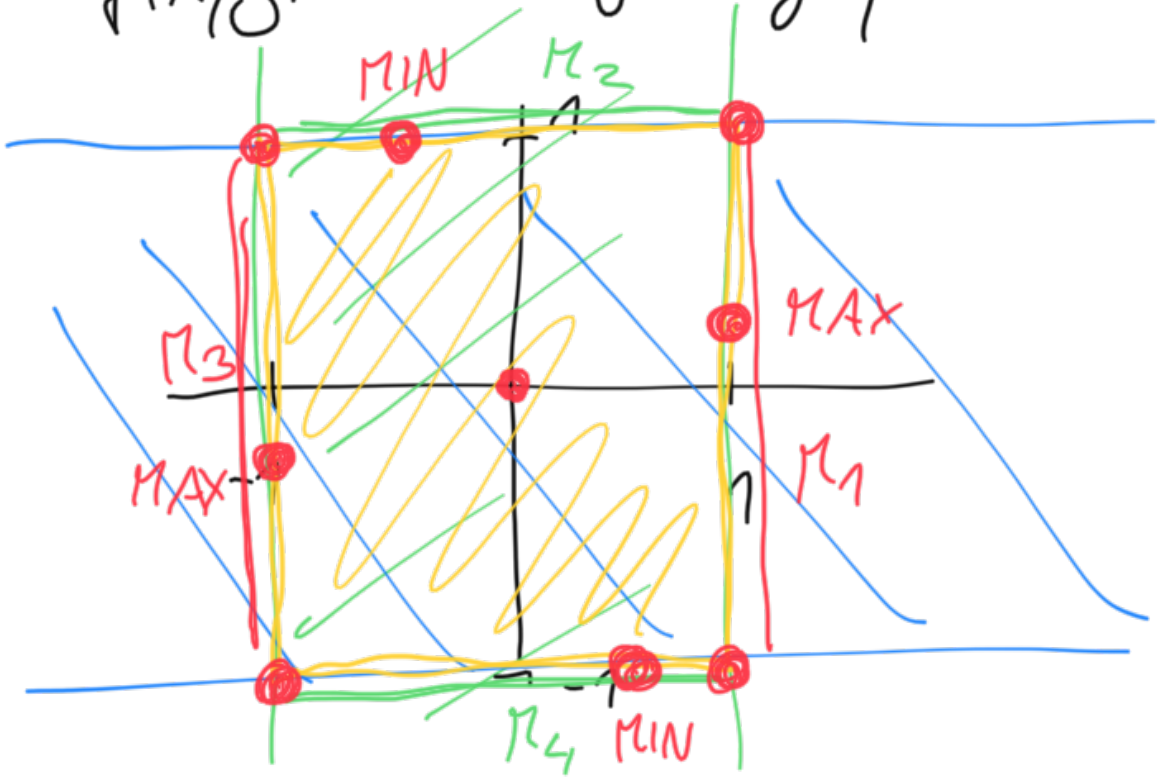
$85 > 9$

$$\frac{}{36} \quad \frac{}{4}$$

$4 \cdot 85 > 9 \cdot 36$ ✓

$340 > 315$

$f(x,y) = x^2 - 3y^2 + xy$, $M = \{[x,y] \in \mathbb{R}^2;\ |x| \leq 1,\ |y| \leq 1\}$



M je omezená, uzavřená (průnik 2 uzav.)

$\Rightarrow$ kompaktní

f je spojitá (na $\mathbb{R}^2$) $\Big\}$ f nabývá na M max. i min

$M = \operatorname{Int} M \cup M_1 \cup M_2 \cup M_3 \cup M_4$

$\operatorname{Int} M = \{|x| < 1, |y| < 1\} = (-1,1) \times (-1,1) = (-1,1)^2$

$M_1 = \{[1,y];\ y \in \langle -1,1 \rangle\}$

$M_2 = \{[x,1];\ x \in \langle -1,1 \rangle\}$

$M_3 = \{[-1,y];\ y \in \langle -1,1 \rangle\}$

$M_4 = \{[x,-1];\ x \in \langle -1,1 \rangle\}$

na $\operatorname{Int} M$: $f \in C^1(\mathbb{R}^2)$

$\nabla f(x,y) = [2x + y,\ -6y + x]$

$= 0 \iff 2x + y = 0 \qquad y = -2x$

$x + 12x = 0$

$\boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}}$

Bod podezřelý z extrému.

na M_1: $g(y) = f(1,y) = 1 - 3y^2 + y$, $y \in \langle -1, 1 \rangle$

$g'(y) = -6y + 1 = 0 \iff y = \frac{1}{6} \in \langle -1, 1 \rangle$

Pod. body: $[1, -1]$
$[1, 1]$
$[1, \frac{1}{6}]$

na M_2: $g(x) \overset{= f(x,1)}{=} x^2 - 3 + x$, $x \in \langle -1, 1 \rangle$

$g'(x) = 2x + 1 = 0 \iff x = -\frac{1}{2}$

Pod. body: $[-1, 1]$
$[1, 1]$
$[-\frac{1}{2}, 1]$

na M_3: $g(y) = f(-1, y) = 1 - 3y^2 - y$, $y \in \langle -1, 1 \rangle$

$g'(y) = -6y - 1 = 0 \iff y = -\frac{1}{6}$

P. body: $[-1; 1]$
$[-1, 1]$
$[-1, -\frac{1}{6}]$

na M_4: $g(x) = f(x, -1) = x^2 - 3 - x$, $x \in \langle -1, 1 \rangle$

$g'(x) = 2x - 1 = 0 \iff x = \frac{1}{2}$

P. b. $[-1, -1]$
$[1, -1]$

$[\frac{1}{2}, -1]$

Dosadíme:

$f(0,0)=0$

$f(1,1)=f(-1,-1)=1-3+1=-1$

$f(-1,1)=f(1,-1)=1-3-1=-3$

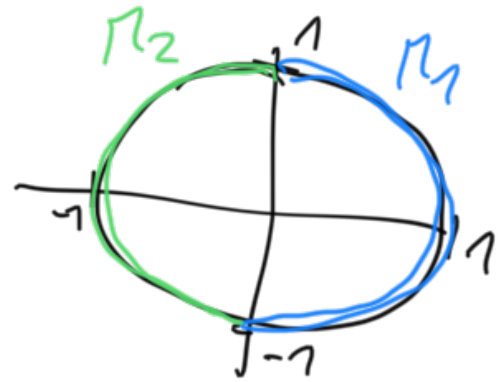
$f(\frac{1}{2},-1)=f(-\frac{1}{2},1)=\frac{1}{4}-3-\frac{1}{2}=-\frac{13}{4}$ min

$f(1,\frac{1}{6})=f(-1,-\frac{1}{6})=1-\frac{3}{36}+\frac{1}{6}=1-\frac{1}{12}+\frac{2}{12}=\frac{13}{12}$ max

---

$f(x,y)=3x^2+4y^3$

$M=\{x^2+y^2=1\}$



a) $M_1=\{y=\sqrt{1-x^2},\ x\in\langle -1,1\rangle\}$

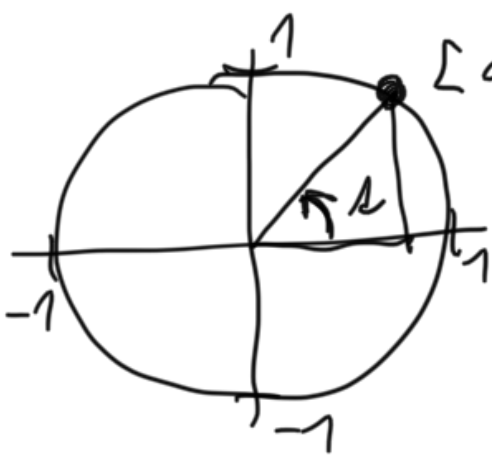
$M_2=\{y=-\sqrt{1-x^2},\ x\in\langle -1,1\rangle\}$

b) $M_1=\{x=\sqrt{1-y^2},\ y\in\langle -1,1\rangle\}$

$M_2=\{x=-\sqrt{1-y^2},\ y\in\langle -1,1\rangle\}$

$\vdots$

c)



$M=\{[\cos t, \sin t];\ t\in\langle 0,2\pi\rangle\}$

$g(t)=f(\cos t,\sin t)=3\cos^2 t+4\sin^3 t,\ t\in\langle 0,2\pi\rangle$

$$g'(t) = 6\cos t \cdot (-\sin t) + 12\sin^2 t \cdot \cos t =$$

$$= 6 \sin t \cos t \, (\underline{2\sin t - 1}) = 0 \iff \boxed{\begin{array}{lll} \sin = 0 & \cos = 0 & \sin = \frac{1}{2} \\ t = 0 & t = \frac{\pi}{2} & t = \frac{\pi}{6} \\ t = \pi & t = \frac{3}{2}\pi & t = \frac{5}{6}\pi \\ t = 2\pi & & \end{array}}$$