$$\sum_{n=1}^{\infty} \underbrace{\frac{(n+5)!\,(2n+1)!\,(-7)^n}{(3n)!}}_{a_n}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\dfrac{(n+6)!\,(2n+3)!\,7^{n+1}}{(3n+3)!}}{\dfrac{(n+5)!\,(2n+1)!\,7^n}{(3n)!}} = \frac{(n+6)(2n+3)(2n+2)\cdot 7}{(3n+3)(3n+2)(3n+1)} \to \frac{28}{27} > 1$$

$\Rightarrow \sum a_n$ D dle podílového krit.

$$\sum_{n=1}^{\infty} \overbrace{\frac{(3n)!}{(2n+700)!\,n!\,(-8)^n}}^{a_n}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\dfrac{(3n+3)!}{(2n+702)!\,(n+1)!\,8^{n+1}}}{\dfrac{(3n)!}{(2n+700)!\,n!\,8^n}} = \frac{(3n+3)(3n+2)(3n+1)}{(2n+702)(2n+701)(n+1)8}$$

$$\downarrow$$

$$\frac{3}{2}\cdot\frac{3}{2}\cdot 3\cdot\frac{1}{8} = \frac{27}{32} < 1$$

$\Rightarrow \sum a_n$ AK dle podílového krit.

$$\sum_{n=1}^{\infty} \underbrace{\frac{(n!)^3}{(3n)!}}_{a_n \geq 0 \text{ .... řada s nezáp. členy}} \log n \qquad \text{K (podílové krit.)}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{((n+1)!)^3}{(3n+3)!}\log(n+1)}{\frac{(n!)^3}{(3n)!}\log n} = \boxed{\frac{(n+1)^3}{(3n+3)(3n+2)(3n+1)}} \cdot \frac{\log(n+1)}{\log n} \to \frac{1}{27} < 1$$

$$\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \frac{1}{27}$$

$$\lim \frac{\log(n+1)}{\log n} = \lim \frac{\log(n(1+\frac{1}{n}))}{\log n} = \lim \frac{\log n + \log(1+\frac{1}{n})}{\log n} =$$

$$= \lim \left(1 + \frac{\log(\overbrace{1+\frac{1}{n}}^{\to 1})}{\log n}\right) \overset{AL}{=} 1 + \frac{0}{+\infty} = 1$$

$$\sum_{n=1}^{\infty} \overbrace{\Big(1 - \underbrace{\sin\frac{1}{n}}_{\in(0,1)}\Big)^{n^2}}^{a_n} \qquad \text{K dle odmoc. krit.}$$

řada s nezáp. členy

$$\sqrt[n]{a_n} = \left(1 - \sin\frac{1}{n}\right)^n \to \frac{1}{e} < 1$$

$$\lim \left(1-\sin\frac{1}{n}\right)^{n} \underset{\substack{\text{HEINE}\\ x_n=\frac{1}{n}}}{=} \lim_{x\to 0+} (1-\sin x)^{\frac{1}{x}} = \lim_{x\to 0+} \exp\left(\frac{1}{x}\cdot\log(1-\sin x)\right) = \frac{1}{e}$$

$$\lim_{x\to 0+} \frac{1}{x}\log(1-\sin x) = \lim_{x\to 0+} \underset{\to 1}{\boxed{\frac{\log(1-\sin x)}{-\sin x}}}\cdot\frac{(-\underset{\to 1}{\boxed{\sin x)}}}{\boxed{x}} = -1$$

$$\sum_{n=1}^{\infty} \underbrace{\frac{1+\sqrt[3]{n}}{1+\sqrt{n^3}}}_{a_n\geq 0}$$

srovnáme s řadou $\sum b_n$, $b_n = \dfrac{\sqrt[3]{n}}{\sqrt{n^3}} = \dfrac{n^{\frac{1}{3}}}{n^{\frac{3}{2}}} = \dfrac{1}{n^{\frac{7}{6}}}$

$\frac{7}{6} > 1 \Rightarrow \sum b_n$ K

$$\lim \frac{a_n}{b_n} = \lim \frac{\frac{1+\sqrt[3]{n}}{1+\sqrt{n^3}}}{\frac{1}{n^{7/6}}} = \lim \frac{1+\sqrt[3]{n}}{\sqrt[3]{n}}\cdot\frac{\sqrt{n^3}}{1+\sqrt{n^3}} \to 1\cdot 1 = 1 \in (0,+\infty)$$

lim. sr. krit. $\sum a_n$ K.