$$\sum_{n=1}^{\infty} \underbrace{\frac{(n+2)^n}{n^{n+1}}}_{a_n \geq 0} \quad D$$

$$a_n = \underbrace{\frac{1}{n} \cdot \left(\underbrace{\overbrace{\frac{n+2}{n}}^{>1}}_{>1}\right)^n} \quad \Rightarrow \quad a_n > \frac{1}{n}$$

srovn. krit.

$$\sqrt[n]{a_n} = \frac{1}{\sqrt[n]{n}} \cdot \frac{n+2}{n} \longrightarrow 1 \quad \text{... nevíme nic}$$

$$\mathcal{D} \sum \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \underbrace{\frac{(n+2)^n}{n^{n+2}}}_{a_n \geq 0} \quad K$$

$$a_n = \frac{1}{n^2} \underbrace{\left(\frac{n+2}{n}\right)^n} \qquad a_n > \frac{1}{n^2} \ \text{\& nic víc}$$

$$\parallel$$

$$\left(1 + \frac{2}{n}\right)^n$$

$$\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^n \underset{\text{HEINE}}{=} \lim_{x\to+\infty} \exp\left(x \cdot \lg\left(1+\frac{2}{x}\right)\right) = e^2$$

$$\lim_{x\to+\infty} x \lg\left(1+\frac{2}{x}\right) = \lim \cancel{x} \frac{\lg\left(1+\frac{2}{x}\right)}{\frac{2}{x}} \cdot \frac{2}{\cancel{x}} =$$

$$= 2 \lim \frac{\lg\left(1+\frac{2}{x}\right)}{\frac{2}{x}} \overset{\text{VOLSF}}{=} 2 \cdot 1 = 2$$

$\frac{\cdot}{x} \to 0$

$\sum a_n$ se „chová jako" $\sum \frac{e^2}{n^2}$

$\Rightarrow$ srovnám s $\sum b_n$, $b_n = \frac{1}{n^2}$, což je konvergentní řada

$$\lim \frac{a_n}{b_n} = \lim \frac{\frac{1}{n^2}\left(\frac{n+2}{n}\right)^n}{\frac{1}{n^2}} = \lim \left(1+\frac{2}{n}\right)^n = e^2 \in (0,+\infty)$$

$$\underset{LSK}{\Rightarrow} \quad \sum a_n \; K \Leftrightarrow \sum b_n \; K$$

$$\sum_{n=1}^{\infty} \underbrace{\left(\sqrt{n^3+n} - \sqrt{n^3-1}\right)}_{a_n \geq 0} \qquad D$$

$$a_n = a_n \cdot \frac{\sqrt{n^3+n}+\sqrt{n^3-1}}{\sqrt{n^3+n}+\sqrt{n^3-1}} = \frac{n^3+n-(n^3-1)}{\sqrt{n^3+n}+\sqrt{n^3-1}} = \frac{n+1}{\sqrt{n^3+n}+\sqrt{n^3-1}}$$

srovnáme s $b_n = \frac{n}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}$:

$$\lim \frac{a_n}{b_n} = \lim \frac{\dfrac{n+1}{\sqrt{n^3+n}+\sqrt{n^3-1}}}{\dfrac{n}{\sqrt{n^3}}} = \lim \frac{1+\frac{1}{n}}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{1}{n^3}}} = \underset{(0,+\infty)}{\frac{1}{2}}$$

$$\overset{\Rightarrow}{\text{LSK}} \quad \sum a_n \ K \Leftrightarrow \sum b_n \ K,$$

ale $\sum b_n$ D

$$\sum_{n=1}^{\infty} \underbrace{\frac{1}{n^2 \cdot \log(n+1)}}_{a_n \geq 0} \quad K \qquad a_n \leq \frac{1}{n^2}, \quad \sum \frac{1}{n^2} \ K$$

$$\overset{\Rightarrow}{\text{sr. krit.}} \quad \sum a_n \ K$$

$$\sum_{n=2}^{\infty} \underbrace{\frac{1}{n^2-1} \cdot \log \frac{1}{n}}_{a_n} \quad AK$$

Pozor, $\forall n \geq 2$ $a_n < 0$.

$$a_n = -\frac{\log n}{n^2-1}, \quad |a_n| \geq \frac{1}{n^2}, \text{ na nic}$$

$|a_n| = \log n$

$$|a_n| = \frac{\log n}{n^2-1}$$

$$\frac{\log n}{n^2} = \frac{\log n}{n^{\frac{3}{2}} \cdot n^{\frac{1}{2}}}$$

růst. škála: „$\log n << n^{\frac{1}{2}}$", neboli $\lim_{n\to\infty} \frac{\log n}{n^{\frac{1}{2}}} = 0$

Srovnám s $\sum \underbrace{\frac{1}{n^{\frac{3}{2}}}}_{b_n}$, což je K řada.

LSK: $$\lim \frac{|a_n|}{b_n} = \lim \frac{\frac{\log n}{n^2-1}}{\frac{1}{n^{\frac{3}{2}}}} = \lim \underbrace{\frac{\log n}{n^{\frac{1}{2}}}}_{\to 0} \cdot \underbrace{\frac{1}{1-\frac{1}{n^2}}}_{\to 1} = 0$$

$$\sum b_n \text{ K} \Rightarrow \sum |a_n| \text{ K}$$

Závěr: $\sum a_n$ AK

$$\sum_{n=1}^{\infty} \underbrace{n \cdot \sin\big(\overbrace{1-\cos\tfrac{1}{n}}^{\to 0}\big)}_{a_n} \quad \text{D}$$

$\sin x$ v 0 se „chová jako" x, tedy $\sin\left(1-\cos\frac{1}{n}\right)$ se „chová jako" $1-\cos\frac{1}{n}$

$0 \le \cos\frac{1}{n} < 1 \Rightarrow \boxed{a_n \ge 0}$

$1-\cos x$ v 0 se „chová jako" x^2,

tedy $1-\cos\frac{1}{n}$ se „chová jako" $\frac{1}{n^2}$

LSK s $b_n = \frac{1}{n}$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin\left(1-\cos\frac{1}{n}\right)}{\frac{1}{n^2}} \underset{\text{HEINE}}{=} \lim_{x\to 0} \frac{\sin(1-\cos x)}{x^2} =$$

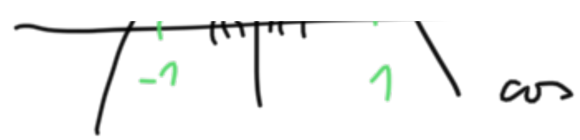
$$= \lim \frac{\sin(1-\cos x)}{1-\cos x} \cdot \frac{1-\cos x}{x^2} = 1 \cdot \frac{1}{2} = \frac{1}{2} \in (0,+\infty)$$

$\Rightarrow \sum a_n \; K \Leftrightarrow \sum b_n \; K$, ale $\sum b_n \; D$

$$\sum_{n=1}^{\infty} \underbrace{\log \cos \overbrace{\frac{(-1)^n}{\sqrt{n}}}^{\to 0}}_{a_n}$$

$\in \langle -1, 1\rangle$

$\cos\frac{(-1)^n}{\sqrt{n}} \in (0,1) \Rightarrow \boxed{a_n < 0}$



$$\boxed{0 \le -a_n} = -\log\left(\cos\left(\underbrace{\frac{1}{\sqrt{n}}}_{\to 0}\right)\right)\quad \cos\frac{1}{\sqrt{n}} \to 1$$

cos je sudá, takže

$$\cos -\frac{1}{\sqrt{n}} = \cos\frac{1}{\sqrt{n}}$$

$\log x$ v 1 se „chová jako" $x-1$

$-\log\cos\frac{1}{\sqrt{n}}$ se „chová jako" $1-\cos\frac{1}{\sqrt{n}}$, což se „chová jako" $\frac{1}{n}$ $(n\to\infty)$

LSK: srovnám s $\frac{1}{n}$

$$\lim \frac{-a_n}{\frac{1}{n}} = \lim \frac{-\log\cos\frac{1}{\sqrt{n}}}{\frac{1}{n}} = \lim \frac{\log\cos\frac{1}{\sqrt{n}}}{\cos\frac{1}{\sqrt{n}}-1}\cdot\frac{1-\cos\frac{1}{\sqrt{n}}}{\frac{1}{n}} \overset{\text{(HEINE VOSF)}}{\underset{\text{AL}}{=}} 1\cdot\frac{1}{2} \in (0,+\infty)$$

$\Rightarrow \sum -a_n$ K $\Leftrightarrow \sum \frac{1}{n}$ K, ale ta diverguje

$\Rightarrow \sum -a_n$ D $\Rightarrow \sum a_n$ D

$$\sum_{n=1}^{\infty} \overbrace{(-1)^n \frac{n+1}{\underbrace{(n+1)\sqrt{n+1}-1}_{a_n \ge 0}}}^{c_n}$$

.... alternující řada

AK: $|c_n| = a_n$ LSK s $b_n = \frac{1}{\sqrt{n}}$

$$\lim \frac{|c_n|}{b_n} = \lim \frac{a_n}{b_n} = \lim \frac{(n+1)\cdot\sqrt{n}}{(n+1)\sqrt{n+1}-1} = 1 \in (0,+\infty)$$

$\sum b_n$ D $\Rightarrow$ $\sum c_n$ nekonverguje absolutně

NAK: nutná podm.: $\lim a_n = \dots = 0$

LEIBNIZ 1) ✓

2) je $\{a_n\}$ monotónní?

$$a_n = \frac{n+1}{(n+1)\sqrt{n+1}-1} \overset{?}{\geq} \frac{n+2}{(n+2)\sqrt{n+2}-1} = a_{n+1}$$

$$(n+1)\left((n+2)\sqrt{n+2}-1\right) \geq (n+2)\left((n+1)\sqrt{n+1}-1\right)$$

$$(n+1)(n+2)\sqrt{n+2} \cancel{-n}-1 \geq (n+2)(n+1)\sqrt{n+1} \cancel{-n}-2$$

$$(n+1)(n+2)\left(\sqrt{n+2}-\sqrt{n+1}\right) \geq -1$$

OK

$\underbrace{\qquad\qquad}_{\geq 0}$

LEIBNIZ

$\Rightarrow \sum c_n$ konverguje neabsolutně

$$\sum_{n=1}^{\infty} \overbrace{(-1)^n \underbrace{\left(\sqrt[3]{n^2-n} - \sqrt[3]{n^2-2n} \right)}_{a_n \geq 0}}^{c_n}$$

$$|c_n| = a_n = \frac{n^2 - n - n^2 + 2n}{\left(\sqrt[3]{n^2-n}\right)^2 + \sqrt[3]{n^2-n}\,\sqrt[3]{n^2-2n} + \left(\sqrt[3]{n^2-2n}\right)^2} = \frac{n}{\sim}$$

pro $n > 1$

…, chová se jako $\frac{1}{n^{\frac{1}{3}}}$

$$\lim \frac{|c_n|}{\frac{1}{n^{\frac{1}{3}}}} = \lim \frac{a_n}{\frac{1}{n^3}} = \ldots = \frac{1}{3}$$

LSK : $\sum |c_n|$ D $\left(\sum \frac{1}{n^{\frac{1}{3}}} \text{ D} \right)$

LEIBNIZ …

Ověříme: 1) $\lim a_n = 0$

2) monotonii $\{a_n\}$:

$$a_n = \frac{n}{n^{\frac{4}{3}}\left(\left(\sqrt[3]{1-\frac{1}{n}}\right)^2 + \sqrt[3]{1-\frac{1}{n}}\sqrt[3]{1-\frac{2}{n}} + \left(\sqrt[3]{1-\frac{2}{n}}\right)^2\right)} =$$

$$= \frac{1}{\underbrace{\underbrace{n^{\frac{1}{3}}}_{\text{rost. } \geq 0}\underbrace{\left(\underbrace{\left(\underbrace{\sqrt[3]{1-\frac{1}{n}}}_{\text{rost. } \geq 0}\right)^2}_{\text{rost.}} + \underbrace{\underbrace{\sqrt[3]{1-\frac{1}{n}}}_{\text{rost. } \geq 0}\underbrace{\sqrt[3]{1-\frac{2}{n}}}_{\text{rost. } \geq 0}}_{\text{rost.}} + \underbrace{\left(\underbrace{\sqrt[3]{1-\frac{2}{n}}}_{\text{rost. } \geq 0}\right)^2}_{\text{rost.}}\right)}}_{\text{rost.}}}$$

$\Rightarrow \{a_n\}$ klesající