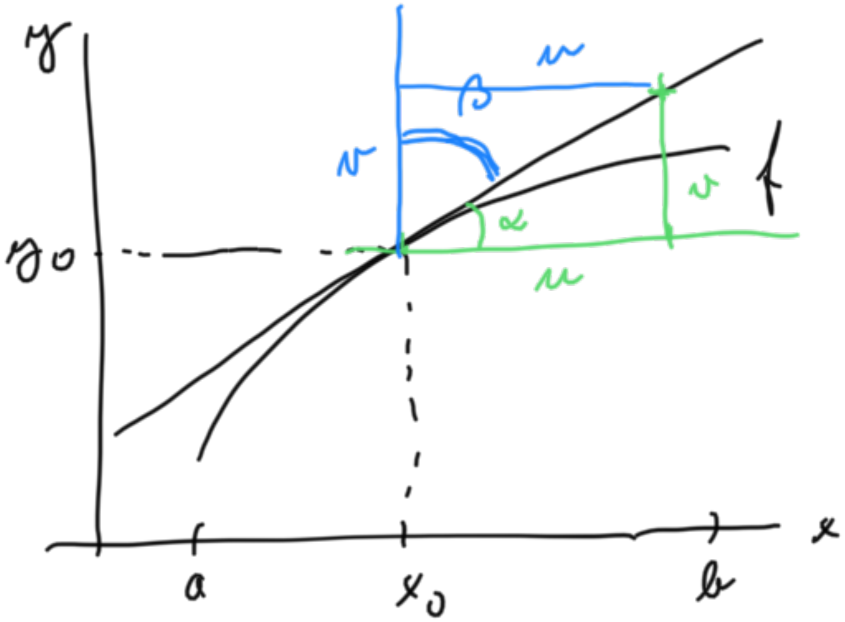


$tg\,\alpha = \frac{v}{u} = f'(x_0)$

$tg\,\beta = \frac{u}{v} = (f^{-1})'(y_0) = \frac{1}{\frac{v}{u}} = \frac{1}{f'(x_0)}$

Důkaz:

$D_{f^{-1}} = f((a,b))$ ... interval (V 30)

x_0 je vnitřní bod (a,b), f je ryze monotónní $\Rightarrow$

$y_0 = f(x_0)$ je vnitřním bodem intervalu $f(a,b)$

$f^{-1}(y_0) = x_0$

... musíme počítat oboustrannou derivaci $(f^{-1})'(y_0) =$

$$= \lim_{y\to y_0} \underbrace{\frac{f^{-1}(y)-f^{-1}(y_0)}{y-y_0}}_{y\in f(a,b)\setminus\{y_0\}} = \lim_{y\to y_0} (h\circ f^{-1})(y) = \frac{1}{f'(x_0)}$$

$h(x) = \frac{x-x_0}{f(x)-f(x_0)}$, $x\in(a,b)\setminus\{x_0\}$

$\lim h(x) \overset{AL}{=} \frac{1}{f'(x_0)}$

VOLSE: ...

vnitřní f^{-1} | $\lim\limits_{y\to y_0} f^{-1}(y) \underset{\uparrow}{=} f^{-1}(y_0) = x_0$

f^{-1} je spojitá (V33)

(P) ..... f^{-1} je prostá (ryze monotónní)

$\square$

$n \in \mathbb{Z},\ n<0$

$$x^n = \frac{1}{x^{-n}} \quad -n>0$$

$$(x^n)' = \frac{0\cdot x^{-n} - 1\cdot(-n)\cdot x^{-n-1}}{(x^{-n})^2} = n\cdot x^{n-1}$$

$$1 = \lim_{x\to 1} \frac{\log x}{x-1} = \lim_{x\to 1}\frac{\log x - \log 1}{x-1} \underset{\uparrow}{=} \log'(1)$$

def. derivace

$x\in(0,+\infty)$

$$(\log)'(x) = \lim_{h\to 0}\frac{\log(x+h)-\log x}{h} = \lim_{h\to 0}\frac{\log\frac{x+h}{x}}{h} = \lim_{h\to 0}\frac{\log\left(1+\frac{h}{x}\right)}{h} =$$

$$= \lim_{h \to 0} \frac{\log\left(1+\frac{h}{x}\right)}{\frac{h}{x}} \cdot \frac{1}{x} \underset{\text{VOLSF}}{=} 1 \cdot \frac{1}{x} = \frac{1}{x}$$

afinní substituce.

$$1 = \lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{e^x - e^0}{x - 0} = \exp'(0)$$

$$(\underbrace{\exp x}_{f^{-1}})' \overset{V39}{=} \frac{1}{\underbrace{\log'}_{f}(\underbrace{\exp x}_{f^{-1}(x)})} = \frac{1}{\frac{1}{\exp x}} = \exp x$$

$$(x^a)' = (\underbrace{\exp}_{f}(\underbrace{a \cdot \log x}_{g})))' = (f \circ g)'(x) \overset{V38}{=} f'(g(x)) \cdot g'(x) =$$

$$= \exp(g(x)) \cdot \frac{a}{x} =$$

$$f'(y) = (\exp y)' = \exp y$$

$$y = g(x)$$

$$g'(x) = a \cdot (\log x)' = \frac{a}{x}$$

$$= \exp(a \cdot \log x) \cdot \frac{a}{x} =$$

$$= x^a \cdot \frac{a}{x} =$$

$$= a \cdot x^{a-1}$$

V37(ii)

$$(a^x)' = \big(\exp(\underbrace{x \cdot \log a})\big)' \overset{V38}{=} \exp(x \cdot \log a) \cdot 1 \cdot \log a =$$

$$= a^x \cdot \log a$$

$$1 = \lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\sin x - \sin 0}{x - 0} = \sin'(0)$$

$$(\sin x)' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h \to 0} \frac{2 \sin \frac{h}{2} \cos\left(x + \frac{h}{2}\right)}{h} =$$

$$= \lim_{h \to 0} \underbrace{\boxed{\frac{\sin \frac{h}{2}}{\frac{h}{2}}}}_{\downarrow\, 1} \cdot \underbrace{\cos\left(x + \frac{h}{2}\right)}_{\downarrow\, \cos x} \overset{AL}{=} 1 \cdot \cos x = \cos x$$

cos je spoj. fce, VOCSF

$$(\cos x)' = \left(\sin\left(\underbrace{\frac{\pi}{2} - x}\right)\right)' \overset{V38}{=} \cos\left(\frac{\pi}{2} - x\right) \cdot (-1) = -\cos\left(\frac{\pi}{2} - x\right) = -\sin x$$

$$(\operatorname{tg} x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\overset{(\sin x)' \cdot \cos x}{\cos x \cdot \cos x} - \overset{\sin \cdot (\cos x)'}{\sin x \cdot (-\sin x)}}{\ldots} = \frac{\cos^2 x + \sin^2 x}{\ldots} = 1 \ldots$$

$$\left(\cos x\right) \quad \cos^2 x \quad \cos^2 x \quad = \frac{}{\cos^2 x}$$

$$(\arcsin x)' \overset{V39}{=} \frac{1}{\sin'(\arcsin x)} = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-x^2}}$$

$x \in (-1,1) \Rightarrow \arcsin x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

$$\cos^2 z + \sin^2 z = 1$$

$$\cos^2 z = 1 - \sin^2 z$$

$$|\cos z| = \sqrt{1-\sin^2 z}$$

$z \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \cos z > 0 \Rightarrow \cos z = \sqrt{1-\sin^2 z}$

$$\cos(\arcsin x) = \sqrt{1-\sin^2(\arcsin x)} = \sqrt{1-x^2}$$

$$(\arccos x)' = \left(\frac{\pi}{2} - \arcsin x\right)' = 0 - (\arcsin x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arctg} x)' \overset{V39}{=} \frac{1}{\operatorname{tg}'(\operatorname{arctg} x)} = \frac{1}{\frac{1}{\cos^2(\operatorname{arctg} x)}} = \cos^2(\operatorname{arctg} x) =$$

$$= \frac{1}{1+\operatorname{tg}^2(\operatorname{arctg} x)} = \frac{1}{1+x^2}$$

$$\operatorname{tg}^2 z = \frac{\sin^2 z}{} = \frac{1-\cos^2 z}{} \quad \Big| : \cos^2 z$$

$$\operatorname{tg}^2 x = \frac{\ldots}{\cos^2 x} = \frac{\ldots}{\cos^2 x} \ldots$$

$$\operatorname{tg}^2 x \cdot \cos^2 x = 1 - \cos^2 x$$

$$\cos^2 x \,(1 + \operatorname{tg}^2 x) = 1$$

$$\cos^2 x = \frac{1}{1 + \operatorname{tg}^2 x}$$

$$(\operatorname{arccotg} x)' = \left(\frac{\pi}{2} - \operatorname{arctg} x\right)' = -\frac{1}{1+x^2}$$

$f(x) = \sqrt[n]{x}, \; n \in \mathbb{N}$

n sudé: $D_f = \langle 0, +\infty)$

$$f(x) = x^{\frac{1}{n}}$$

$$f'(x) = \frac{1}{n} \cdot x^{\frac{1}{n} - 1} = \frac{1}{n} \cdot x^{\frac{1-n}{n}} = \frac{1}{n} \sqrt[n]{x^{1-n}}$$

n liché: $D_f = \mathbb{R}$

$x < 0$: $f(x) = -\sqrt[n]{\underbrace{-x}_{>0}} = -g(-x)$

$g(x) = \sqrt[n]{y}, \; y > 0$ $\quad | \quad f'(x) = -\left(g(\underbrace{-x}_{})\right)' =$

"
$\frac{1}{n}$

$$g'(y) = \frac{1}{n}\sqrt[n]{y^{1-n}}$$

vnitřní fce

$$= -g'(-x)\cdot(-1) =$$

$$= g'(-x) = \frac{1}{n}\sqrt[n]{(-x)^{1-n}} =$$

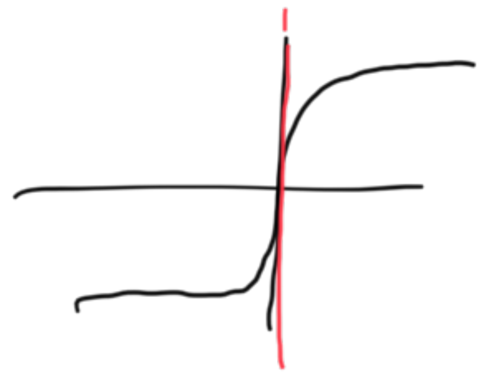
sudé

$$= \frac{1}{n}\sqrt[n]{x^{1-n}}$$

<u>n liché</u>

<u>Dohromady</u>: $f'(x) = \frac{1}{n}\sqrt[n]{x^{1-n}}$, $x \in \mathbb{R}$, $x \neq 0$

$f'(0) = +\infty$ (později)



<u>Důk.</u>: Sporem: nechť $f'(x_0)$ existuje a $f'(x_0) \neq 0$.

a) Předp., že $f'(x_0) > 0$.

$$\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0} > 0 \overset{23(i)}{\Rightarrow} \exists \delta > 0 : \forall x \in P(x_0, \delta) :$$

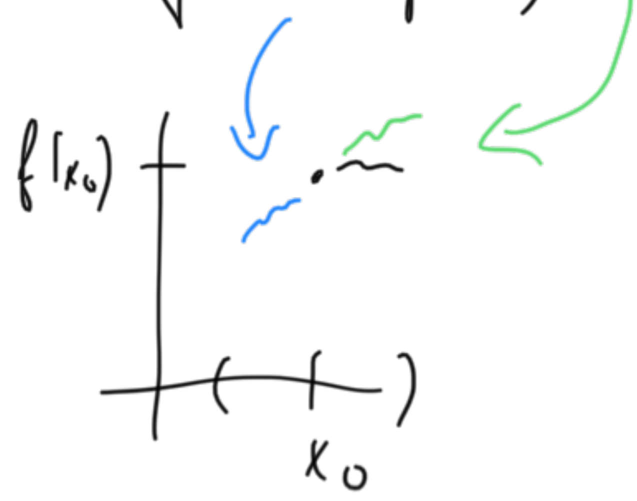
$$\frac{f(x)-f(x_0)}{x-x_0} > 0$$

Tedy pro $x \in (x_0, x_0+\delta)$: $x - x_0 > 0 \Rightarrow f(x) > f(x_0)$

$x \in (x_0 - \delta, x_0) : \; x - x_0 < 0 \;\Rightarrow\; f(x) < f(x_0)$

$\Rightarrow$ Tedy v x_0 <u>není</u> lok. extrém.

Spor.



b) $f'(x_0) < 0$ ..... spor dostaneme analogicky

$\square$