$a \in \mathbb{R}$ ... $|a| = \begin{cases} a & \dots\ a \geq 0 \\ -a & \dots\ a < 0 \end{cases}$

$\forall a \in \mathbb{R}: \quad -|a| \leq a \leq |a|$

$a \geq 0 \qquad -|a| \leq a = |a|$

$a < 0 \qquad |a| = -a$

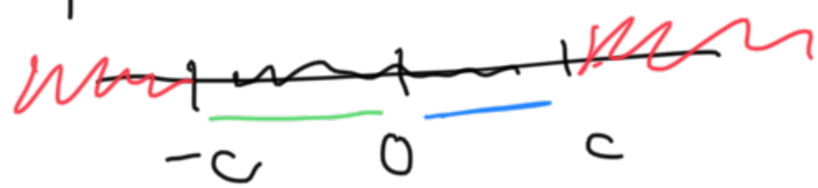
$-|a| = a \leq |a|$

$x, c \in \mathbb{R},\ c > 0$

$|x| < c \iff -c < x < c \qquad \Big|\ x < c \ \&\ x > -c$

$|x| = |x - 0|$

$x \in (-c, c)$



„$\Rightarrow$"

$x \geq 0 \qquad 0 \leq x = |x| < c$

$x < 0 \qquad c > |x| = -x \quad / \cdot (-1)$

$-c < x < 0$

„$\Leftarrow$" $x \geq 0 \qquad 0 \leq x < c$, $x = |x|$, $|x| < c$

$x < 0 \qquad -c < x < 0 \quad / \cdot (-1)$, $x = -|x|$

$|x| < c$

jinak:

$-c < -|x| \leq x \leq |x| < c$

$|x| < c \ / \cdot (-1)$

$-|x| > -c$

$|x| > c \iff (x > c) \vee (x < -c) \qquad \Big|\ |x| \geq c \iff (x \geq c) \vee (x \leq -c)$

$$x \in (-\infty, -c) \cup (c, +\infty) \;\Big|\; \neg\, |x| < c \iff \neg\,(x < c \;\&\; x > -c)$$

$$\neg\,(-c < x < c)$$

$$|x| = c \iff x = c \;\vee\; x = -c$$

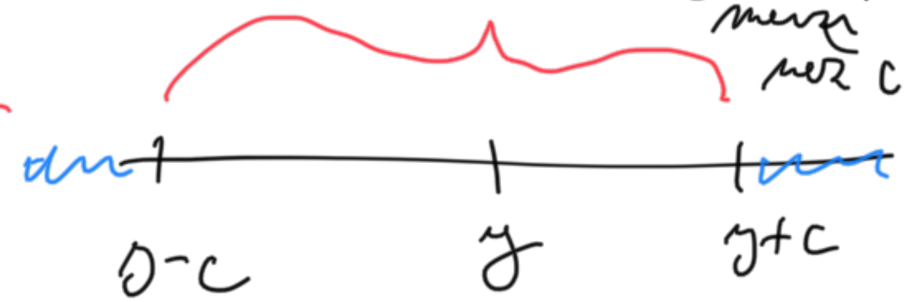
$$|x-y| < c \iff -c < x - y < c \quad /+y \quad \Big|\; -c < x-y \;\&\; x-y < c \quad /+y$$

$$y - c < x \qquad x < y + c$$

$$y - c < x < y + c$$

$$x \in (y-c,\, y+c)$$

„vzdálenost x od y je menší než c“



$$|x-y| \le c \iff x \in \langle y-c,\, y+c \rangle$$

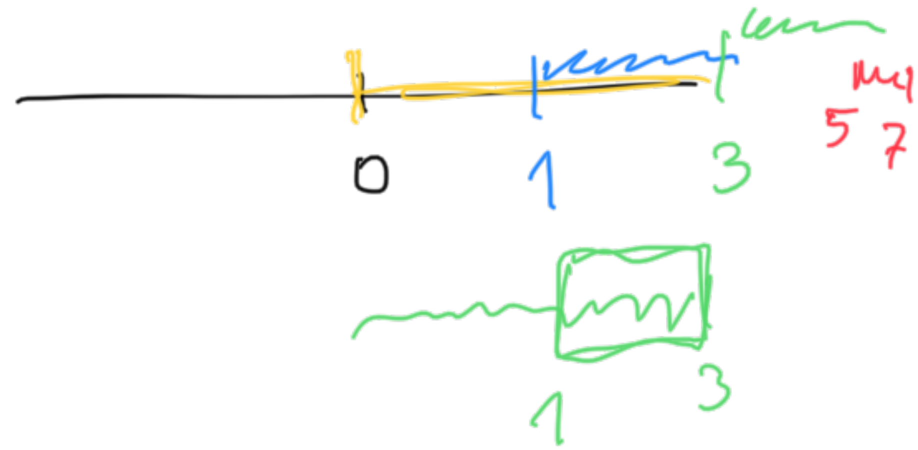
$$|x-y| > c \iff x \in (-\infty,\, y-c) \cup (y+c,\, +\infty)$$

$$|x-y| \ge c \iff x \in (-\infty,\, y-c\rangle \cup \langle y+c,\, +\infty)$$

$$\{x \in \mathbb{R};\ ||||x|-1|-2|-3| < 1\}$$

$x \geq 0$

$$|||x-1|-2|-3|<1$$



$x \geq 1$

$$||x-1-2|-3|<1$$

$$||x-3|-3|<1$$

$x \geq 3$

$$|x-3-3|<1$$

$$|x-6|<1$$

$$x \in (5,7)$$

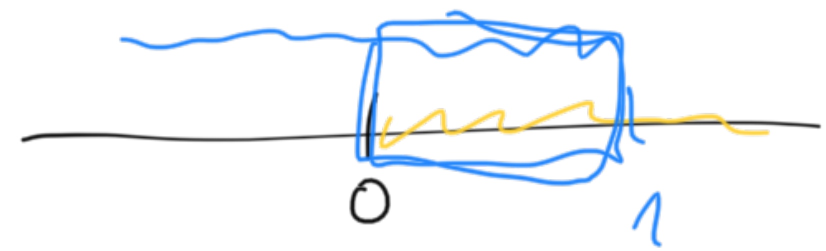
$x<3$

$$|3-x-3|<1$$

$$|-x|<1$$

$$|x|<1$$

$$\emptyset$$



$x<1$

$$||1-x-2|-3|<1$$

$$||-1-x|-3|<1$$

$$||x+1|-3|<1$$

>0

$$|x+1-3|<1$$

$$|x-2|<1$$

$$x\in(1,3)$$

$\emptyset$

$\underline{x<0}$

$$|||-x-1|-2|-3|<1$$

$$|||x+1|-2|-3|<1$$

$$|a|=|-a|$$

$$|-a|=|(-1)\cdot a|=$$
$$=|-1|\cdot|a|=$$
$$=1\cdot|a|=|a|$$

$\underline{x\geq -1}$

$$||x+1-2|-3|<1$$

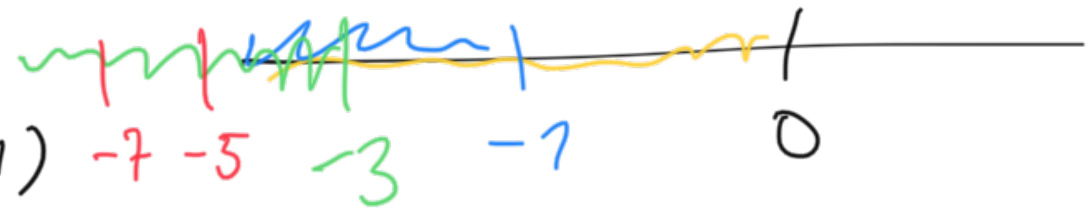
$$||\underbrace{x-1}_{<0}|-3|<1$$

$$|-x+1-3|<1$$

$$|-x-2|<1$$

$$|x+2|<1$$

$$x\in(-3,-1)$$



$\emptyset$

$\underline{x<-1}$

$$||-x-1-2|-3|<1$$

$$||-x-3|-3|<1$$

$$||x+3|-3|<1$$

$x \geq -3$

$$|x+3-3|<1$$

$$|x|<1$$

$$x \in (-1,1)$$

$\emptyset$

$x<-3$

$$|-x-3-3|<1$$

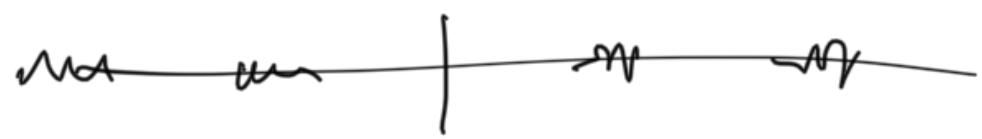
$$|-x-6|<1$$

$$|x+6|<1$$

$$x \in (-7,-5)$$

Celkem: $x \in (-7,-5) \cup (5,7)$

jinak: $|||\underbrace{|x|}_{y}-1|-2|-3|<1$ $\quad y=|x| \geq 0$



$$||\underbrace{|y-1|}_{u}-2|-3|<1$$

$|x|-1\ y \ldots\ldots \quad x \in$

$u = |y-1| \geq 0$

$||\underbrace{u-2|}_{v} - 3| < 1$

$v = |u-2| \geq 0$

$|v-3| < 1$ ✓ OK

$v \in (2,4)$

$2 < |u-2| < 4$

$u \in \cancel{(-\infty,0)} \cup (4,+\infty) \qquad u \in (-2,6)$

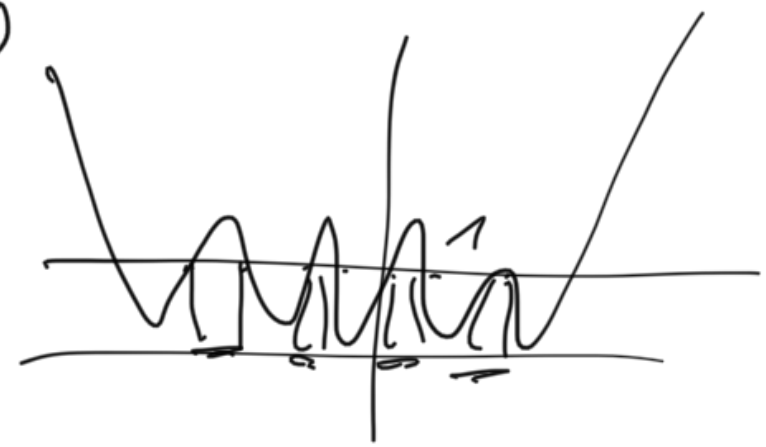
$u \in (4,6)$

$4 < |y-1| < 6$

$y \in \cancel{(-\infty,-3)} \cup (5,+\infty) \qquad y \in (-5,7)$

$y \in (5,7)$

$\quad = |x|$



$5 < |x| < 7$

$x \in (5,7)$

$x \in (-7, -5)$

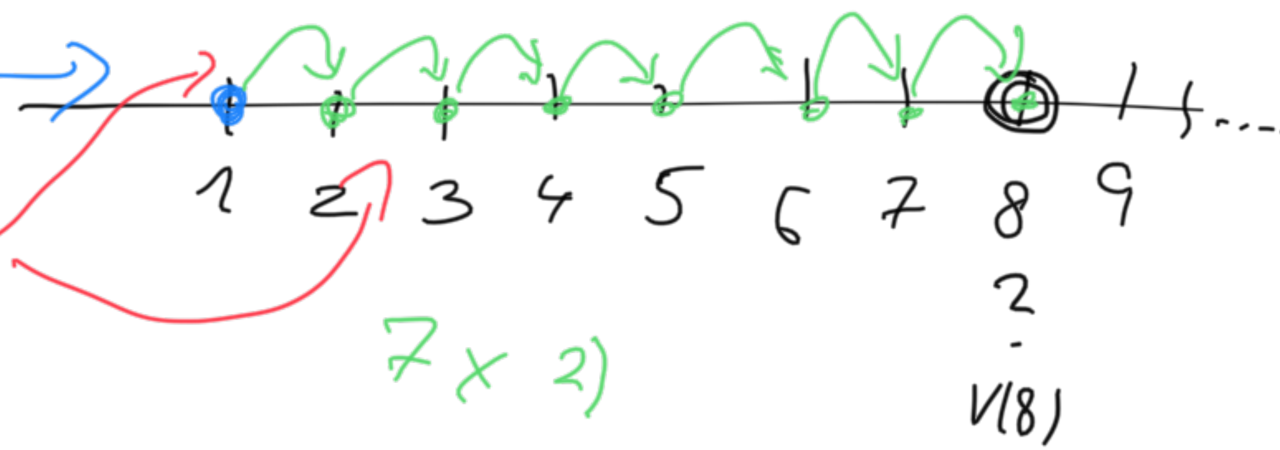
$\forall n \in \mathbb{N} : V(n)$ $\qquad\qquad$ $\forall n \in \mathbb{N} : \ 2n \geq 2$

$V(1), V(2), V(3), \dots$

1) $V(1)$ — první krok

2) $\forall n \in \mathbb{N} : \ V(n) \Rightarrow V(n+1)$ — indukční krok

indukční předpoklad



$$\forall n \in \mathbb{N} : \quad \sum_{k=1}^{n} k^2 = \frac{1}{6} n (n+1)(2n+1)$$

1) $V(1)$: $\quad \sum_{k=1}^{1} k^2 = \frac{1}{6} \cdot 1 \cdot 2 \cdot 3 = 1 \quad \checkmark$

$1 = 1^2$

2) Nechť $n \in \mathbb{N}$ je libovolné. $\qquad$ VÍM

Předpokládejme, že platí $V(n)$: $\quad \sum_{k=1}^{n} k^2 = \frac{1}{6} n (n+1)(2n+1)$

CHCI $V(n+1)$, tj.: $$\sum_{k=1}^{n+1} k^2 = \frac{1}{6}(n+1)(n+2)(2n+3)$$

CHCI

$$\sum_{k=1}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2 \underset{\text{ind. předp.}}{=} \frac{1}{6} n(n+1)(2n+1) + (n+1)^2 =$$

$$= (n+1)\left(\frac{1}{6} n(2n+1) + n+1\right) = (n+1)\left(\frac{1}{3} n^2 + \frac{1}{6} n + n + 1\right) =$$

$$= \frac{1}{6}(n+1)\left(2n^2 + n + 6n + 6\right) = \frac{1}{6}(n+1)\left(2n^2 + 7n + 6\right) =$$

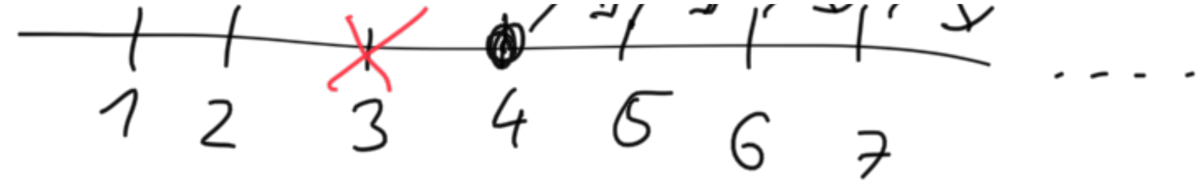
$$\overset{\cancel{?}}{=} \frac{1}{6}(n+1)(n+2)(2n+3)$$

stačí, že $(n+2)(2n+3) = 2n^2 + 7n + 6$

$$\| $$

$$2n^2 + 4n + 3n + 6 \quad \checkmark$$

$\forall n \in \mathbb{N} \setminus \{3\}: \ n^2 \le 2^n$



$V(1)$: $1^2 \le 2^1$, tj: $1 \le 2$ ✓

$V(2)$: $2^2 \le 2^2$ ✓

$V(3)$: $9 = 3^2 \not\le 2^3 = 8$

indukci budu začínat od 4

1) $V(4)$: $16 = 4^2 \le 2^4 = 16$ ✓

2) Nechť $n \in \mathbb{N}$, $n \ge 4$.

Předpokládejme, že $n^2 \le 2^n$. $V(n)$

CHCI: $(n+1)^2 \le 2^{n+1}$ CHCI

$$(n+1)^2 = \underbrace{n^2}_{} + 2n + 1 \underset{\text{ind. předp.}}{\le} 2^n + 2n + 1 \overset{?}{\le} 2^{n+1} = 2 \cdot 2^n$$

stačí ukázat, že $2n+1 \le 2^n$

kdybych věděl, že $2n+1 \le n^2$, jsem hotov

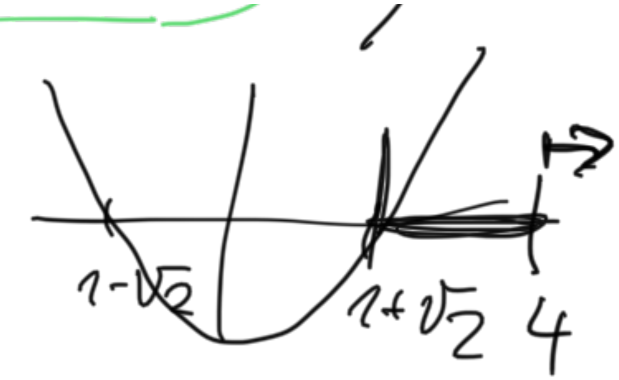
(použiji znovu ind. předp.:

$2n+1 \le n^2 \le 2^n$)

CHCI: $n^2 - 2n - 1 \geq 0$

$n \geq 4$

$D = 4 + 4 = 8$



$$n_{1,2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

platí pro $n \geq 1 \pm \sqrt{2}$ OK