$\forall n, k \in \mathbb{N}$. Pak ex. $q, r \in \mathbb{N} \cup \{0\}$, pro která $r < k$ a $n = qk + r$

(dělení se zbytkem)

Zvolme $k \in \mathbb{N}$ libovolné. CHCI $\forall n \in \mathbb{N}\ \exists q, r \ldots\ldots$

mat. indukcí: 1) $n = 1$ CHCI $1 = q \cdot k + r$

- $k = 1 \ldots\ q = 1,\ r = 0$
- $k > 1 \ldots\ q = 0,\ r = 1$

2) Předpokládejme, že to platí pro $n \in \mathbb{N}$.

VIM: $n = qk + r$, $r < k$

CHCI: $n+1 = q'k + r'$, $r' < k$

$$\left.\begin{aligned} q' &= q \\ r' &= \underbrace{r+1}_{<k} \end{aligned}\right\} n+1 = qk + r + 1$$

- $r + 1 < k$: $(r < k-1)$
- $r = k - 1$: $q' = q + 1$, $r' = 0$

$$n+1 = (q+1)k + r' = qk + k + r' =$$
$$= qk + r + 1 + r' = \underbrace{qk + r}_{n} + 1$$

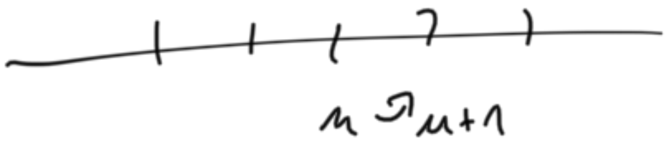
$a_1, a_2, \ldots, a_n \geq 0$, CHCI $\sqrt[n]{\ldots} \leq \frac{a_1 + \ldots + a_n}{\ldots}$

$\sqrt{a_1 \cdot \ldots \cdot a_n} = \frac{\qquad}{n}$

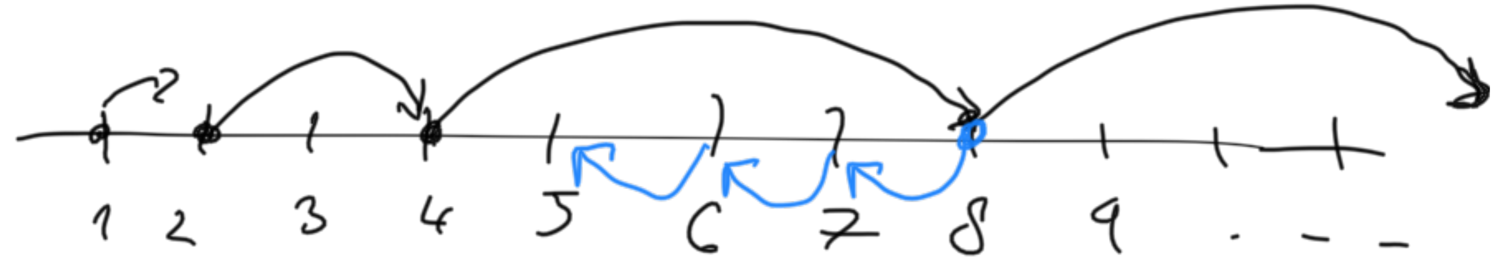
$\sqrt[n]{} \overset{?}{\longrightarrow} \sqrt[n+1]{}$ | $\sqrt[n]{} \longrightarrow \sqrt[2n]{}$

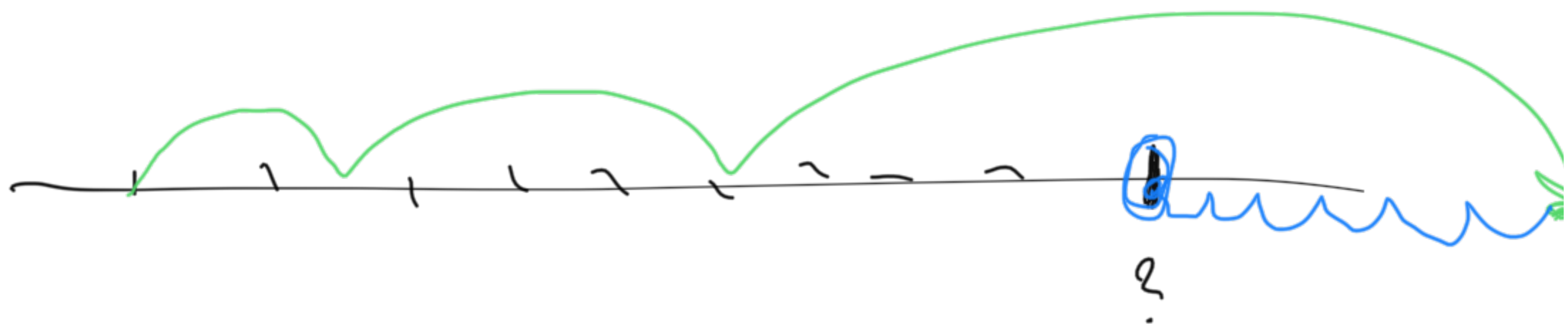
$V(n) \Rightarrow V(2n)$

$V(n+1) \Rightarrow V(n)$



$V(n) \Rightarrow V(n+1)$



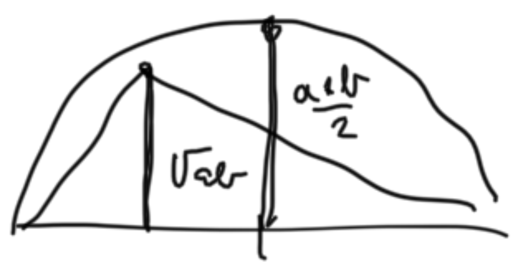


$\underline{n=1}$ $\quad \sqrt[1]{a_1} \leq \frac{a_1}{1} = a_1$

a_1^{1}

$a, b \geq 0$

$\underline{n=2}$ $\quad \sqrt{ab} \leq \frac{a+b}{2}$



$ab \leq \frac{1}{4}(a+b)^2$

$4ab \leq a^2 + 2ab + b^2$

$0 \leq a^2 - 2ab + b^2 = (a-b)^2$ OK

$V(n) \Rightarrow V(2n)$ ...

$V(n) \Rightarrow V(2n)$ Předp., že AG nerovnost platí pro libovolnou n-tici nezáp. čísel.

Nechť $a_1, a_2, \dots, a_{2n}$, CHCI
$$\sqrt[2n]{a_1 \cdots a_{2n}} \le \frac{a_1 + \dots + a_{2n}}{2n}$$
$$\|$$
$$\sqrt{\sqrt[n]{a_1 \cdots a_n} \cdot \sqrt[n]{a_{n+1} \cdots a_{2n}}}$$

Dle ind. předp. $-\,\|\,- \le \sqrt{\underbrace{\frac{a_1 + \dots + a_n}{n}}_{a} \cdot \underbrace{\frac{a_{n+1} \cdots a_{2n}}{n}}_{b}} \le$

$$\le \frac{a+b}{2} = \frac{\frac{a_1 + \dots + a_n}{n} + \frac{a_{n+1} + \dots + a_{2n}}{n}}{2} =$$

$$= \frac{a_1 + \dots + a_{2n}}{2n} \quad \checkmark$$

$V(n+1) \Rightarrow V(n)$ Předpokládejme, že AG ner. platí pro lib. $(n+1)$-tici nezáp. čísel.

Nechť $a_1, \dots, a_n \geq 0$.

Položme $b_i = \dfrac{a_i}{\sqrt[n]{a_1 \cdots a_n}}$, $i = 1, \dots, n$, $b_{n+1} = 1$.

víme: $$\sqrt[n+1]{b_1 \cdots b_{n+1}} \le \frac{b_1 + \dots + b_{n+1}}{n+1}$$

$$\sqrt[n+1]{\underbrace{\frac{a_1}{\sqrt[n]{\cdots}} \cdot \frac{a_2}{\sqrt[n]{}} \cdots \frac{a_n}{\sqrt[n]{}}}_{\overset{\shortparallel}{1}} \cdot 1} \le \frac{\frac{a_1}{\sqrt[n]{}} + \dots + \frac{a_n}{\sqrt[n]{}} + 1}{n+1}$$

$\sqrt[n+1]{} \to \sqrt[n]{}$

$\sqrt[n+1]{1} \to \sqrt[n]{1}$

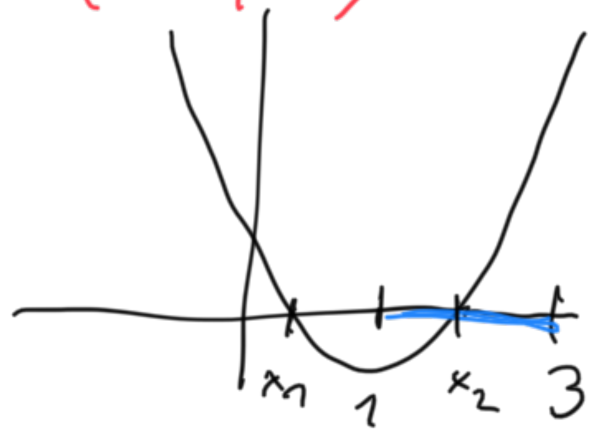
$$1 \le \frac{\frac{a_1}{\sqrt[n]{}} + \dots + \frac{a_n}{\sqrt[n]{}} + 1}{n+1} \quad \Big/ \cdot (n+1)$$

$$n+1 \le \frac{a_1}{\sqrt[n]{}} + \dots + \frac{a_n}{\sqrt[n]{}} + 1$$

$$n \le \frac{a_1 + \dots + a_n}{\sqrt[n]{a_1 \cdots a_n}}$$

$$\sqrt[n]{a_1 \cdots a_n} \le \frac{a_1 + \dots + a_n}{n}$$

$$\{a\in\mathbb{R};\ \underbrace{\forall x\in\mathbb{R}:\ |x-2|\le 1 \Rightarrow x^2-ax>5}\} = (-\infty,-4)$$



$$\forall x\in\mathbb{R},\ |x-2|\le 1:\quad x^2-ax>5$$

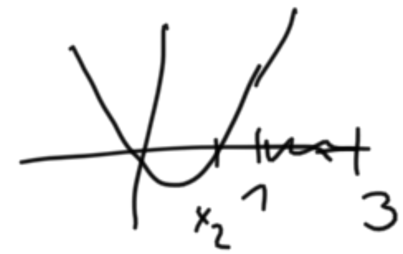
$$\{a\in\mathbb{R};\ \forall x\in\langle 1,3\rangle:\ x^2-ax>5\}$$

Hledám vš. parametry $a\in\mathbb{R}$ takové, že $x^2-ax-5>0$ platí na intervalu $\langle 1,3\rangle$.

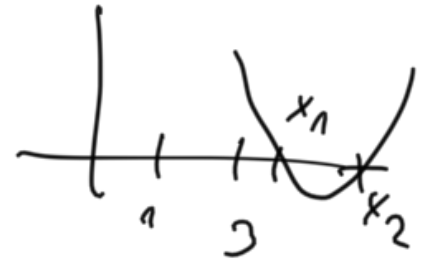
$$D = a^2+20$$

$$x_{1,2} = \frac{a\pm\sqrt{a^2+20}}{2}$$

$x_2<1$



$x_1>3$



$$\frac{a+\sqrt{a^2+20}}{2}<1$$

$$a+\sqrt{a^2+20}<2$$

$$\sqrt{a^2+20}<2-a$$

$2-a\le 0$ $\qquad$ $a<2$

$$\frac{\overbrace{a-\overbrace{\sqrt{a^2+20}}^{>a}}^{<0}}{2}>3$$

$a \geq 2$

$\times$

$\cancel{a^2} + 20 < (2-a)^2 = 4 - 4a + \cancel{a^2}$

$4a < -16$

$\underline{a < -4}$

$\underline{a \in (-\infty, -4)}$

(i) $\forall x \in M \ \exists y \in M \ \exists k \in M : x = y + k$

	(i)		(ii)	(iii)
$M = \mathbb{N}$	ne $x = 1$	$1 = y + k$	ne $x > 1$	ne $x = 1$
$M = \mathbb{N} \cup \{0\}$	ano	zvolíme $y = x, k = 0$	ano $y = 0, k \in \ell$	ne $x = y + k + 1$
$M = (0,1)$	ano	$y = k = \frac{x}{2} \in (0,1)$	ne $x = y$	ne $x = \frac{y}{2} + \frac{k}{2} = \frac{y+k}{2}$
$M = \{0\}$	ano		ano	ano

(ii) $\exists y \in M \ \forall x \in M \ \exists k \in M : x = y + k$

(iii) $\exists y \in M \ \exists k \in M \ \forall x \in M : x = y + k$

$$\exists x \in M\ \forall y \in M\ \forall R \in M:\ x \neq y + R$$

$x \in (\alpha - 1, \alpha + 1)$

$$\forall a \in \mathbb{R}\ \exists \varepsilon > 0\ \exists \alpha \in \mathbb{R}\ \forall x \in \mathbb{R}:\ \big(x \in (a, a+\varepsilon) \Leftrightarrow |x - \alpha| < 1\big)$$

$$(a, a+\varepsilon) = (\alpha - 1, \alpha + 1)$$

$$\Updownarrow$$

$$a = \alpha - 1\ \&\ a + \varepsilon = \alpha + 1$$

$$\forall a \in \mathbb{R}\ \exists \varepsilon > 0\ \exists \alpha \in \mathbb{R}:\ a = \alpha - 1\ \&\ a + \varepsilon = \alpha + 1$$

vezmu $\alpha = a + 1$

$\cancel{\alpha} + \varepsilon = a + 1 + 1 = \cancel{a} + 2$

$\varepsilon = 2 > 0$

ANO

$$\exists a \in \mathbb{R}\ \forall \varepsilon > 0\ \forall \alpha \in \mathbb{R}\ \forall x \in \mathbb{R}:\ \big(x \in (a, a+\varepsilon) \Leftrightarrow |x - \alpha| < 1\big)$$

$$\exists a \in \mathbb{R}\ \forall \varepsilon > 0\ \forall \alpha \in \mathbb{R}:\ a = \alpha - 1\ \&\ a + \varepsilon = \alpha + 1$$

musí být $\varepsilon = 2$, kdežto NE

$$\exists a \in \mathbb{R}\ \forall \varepsilon > 0\ \forall \alpha \in \mathbb{R}\ \boxed{\exists x \in \mathbb{R}:\ \big(x \in (a, a+\varepsilon) \Leftrightarrow |x - \alpha| < 1\big)}$$

$\Leftarrow$ pravidla: $x \in (a, a+\varepsilon)$ & $x \in (\alpha-1, \alpha+1)$

$\vee$

$x \notin (a, a+\varepsilon)$ & $x \notin (\alpha-1, \alpha+1)$

platí vždy nezávisle na hodnotách a, ε, α

ANO

platí vždy stačí vzít

$x = \min\{a, \alpha-1\} - 1$