$$\forall c \in \mathbb{R}, c>0 \quad \underbrace{\forall x \in \mathbb{R}: \ |a-x|<c \Rightarrow |b-x|<c}$$

$$x \in (a-c, a+c) \qquad x \in (b-c, b+c)$$

tj: $(a-c, a+c) \subset (b-c, b+c)$

$$\forall c>0: (a-c, a+c) \subset (b-c, b+c)$$

$$\forall c>0: \quad a-\cancel{c} \geq b-\cancel{c} \ \ \& \ \ a+\cancel{c} \leq b+\cancel{c}$$

$$a \geq b \quad \& \quad a \leq b$$

$$\boxed{a=b}$$

$$\forall c>0 \ \forall x \in \mathbb{R}: \ a-x<c \Rightarrow |b-x|<c$$

$$x \in (a-c, +\infty) \qquad x \in (b-c, b+c)$$

$(a-c, +\infty) \subset (b-c, b+c)$ nemůže nastat

Tedy $\forall a, b \in \mathbb{R}$ výrok neplatí.

$$\forall c>0 \ \forall x \in \mathbb{R}: \ |a-x|<c \Rightarrow b-x<c$$

$$x \in (a-c, a+c) \qquad x \in (b-c, +\infty)$$

$$(a-c, a+c) \subset (b-c, +\infty)$$

$$\forall c>0: \ a-c \geq b-c$$

$$\boxed{a \geq b}$$

$$(a-c, +\infty) \subset (b-c, +\infty) \qquad \boxed{a \geq b}$$

$$\forall x \in \mathbb{R} \ \exists u \in \mathbb{R}, u>0 \ \underbrace{\forall c \in (0,u): |a-x|<c \Rightarrow |b-x|<u}$$

$$\forall c \in (|a-x|, u): |b-x|<u$$

$u \leq |a-x|$ — $\emptyset$, tedy $\forall$ je pravdivá

$u > |a-x|$: $|b-x|<u$

Pokud $a \neq x$, pak $|a-x|>0$, stačí vzít $u=|a-x|$ a výrok je pravdivý.

Pokud $a=x$, stačí vzít $u>|b-x|$ libovolné a výrok je pravdivý.

Každý vztah mezi a a b ([illegible] (tj. $[a,b]$ libovolná)

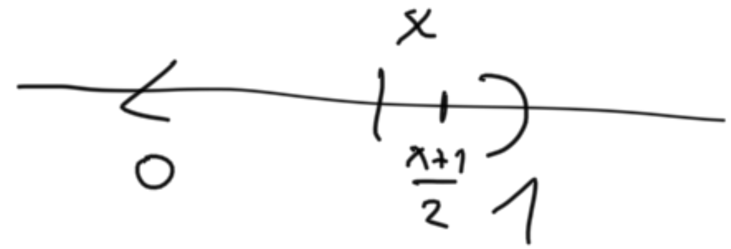
(ve skutečnosti stačí vždy zvolit $n > |b-x|$ a je to)

$A = \langle 0, 1)$

$\min A = 0$
$\quad\shortparallel$
$\inf A$

A nemá maximum:

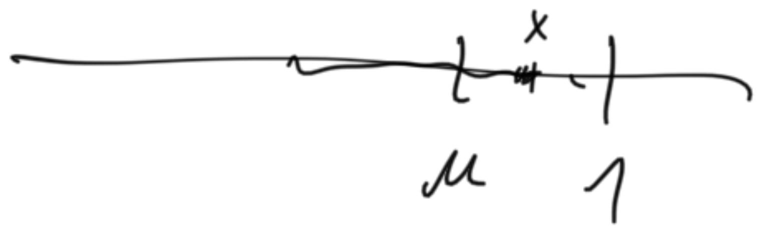
Pro $\forall x \in A$: $x < \frac{x+1}{2} \in A$.



$\sup A = 1 \notin A$ ... A nemá max

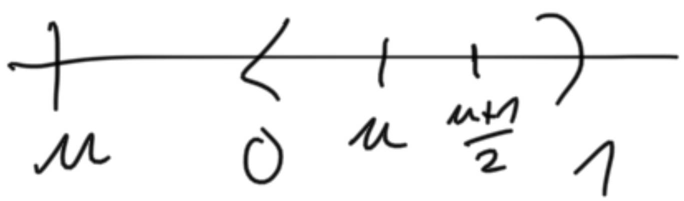
(i) 1 je horní závora A

(ii) nechť $u \in \mathbb{R}$, $u < 1$.



CHCI $\exists x \in A$, $x > u$

$u < 0$, pak zvolme $x = 0 \in A$



$u \geq 0$ zvolme

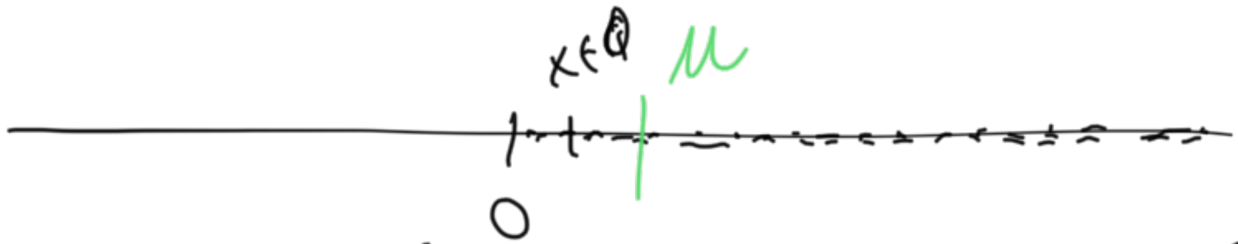
$1 > x = \frac{u+1}{2} > u$

$\in A$

$C = \{1, -5, 7, -3, 50\}$

$\underbrace{-5}_{\text{min (inf)}} < -3 < 1 < 7 < \underbrace{50}_{\text{max (sup)}}$

$B = \{x \in \mathbb{Q},\ x > 0\}$



Z Arch. vl. $\Rightarrow$ B není shora omez. $\Rightarrow$ $\sup B = +\infty$
(nemá max.)

Tvrz: $\inf B = 0$ (nemá min)

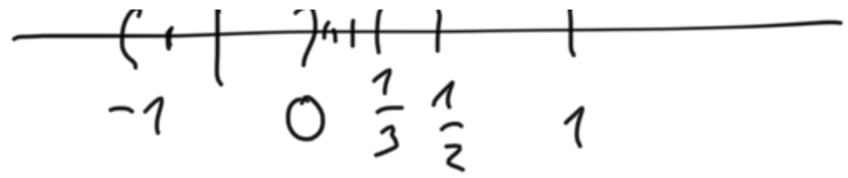
↳ (i) 0 je dolní závora B

(ii) nechť $\varepsilon \in \mathbb{R},\ \varepsilon > 0$

CHCI $\exists x \in \mathbb{Q},\ \underbrace{x > 0 \ \& \ x < \varepsilon}_{x \in (0,\varepsilon)}$

Dle V7 je to pravda.

$D = (-1, 0) \cup \{\frac{1}{n},\ n \in \mathbb{N}\}$



$\max D = 1$

$\hookrightarrow 1 \in D$

$\forall n \in \mathbb{N}: \frac{1}{n} \leq 1$

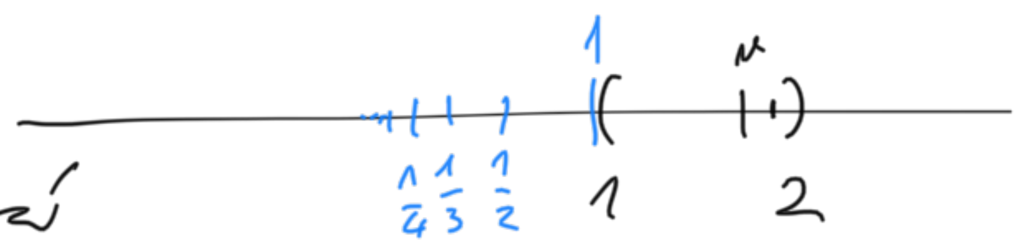
$\forall x \in (-1,0): x < 0 < 1$

$\inf D = -1$ (nemá min)

$$\left.\begin{array}{l} \text{(i)} \ \forall n \in \mathbb{N}: \frac{1}{n} > 0 > -1 \\ \forall x \in (-1,0): x > -1 \end{array}\right\} -1 \text{ je dolní závora}$$

(ii) stejně, jako pro A

$E = (1,2) \cup \{\frac{1}{n}; n \in \mathbb{N}\}$



$\sup E = 2$, max nemá

(i) $\forall n \in \mathbb{N}: \frac{1}{n} \leq 1 < 2$

$\forall x \in (1,2): x < 2$

(ii) stejně, jako A

$\inf E = 0$

(min nemá) (i) $\forall n \in \mathbb{N}: \frac{1}{n} > 0$

$\forall x \in (1,2): x > 1 > 0$

(ii) nechť $u \in \mathbb{R}$, $u > 0$

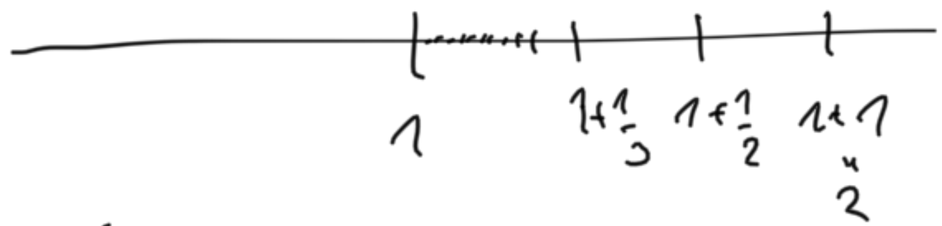
chci $\exists x \in E,\ x < u$

$u > 0$

stačí najít $n \in \mathbb{N}$: $\frac{1}{n} < u \iff n > \frac{1}{u}$

To platí díky arch. vlastnosti.

$$M = \left\{ \frac{n+1}{n},\ n \in \mathbb{N} \right\} = \left\{ 1 + \frac{1}{n},\ n \in \mathbb{N} \right\}$$



$\max M = 2$

$\inf M = 1$, min není

$1 + \frac{1}{n} \leq 2$ | (i) $\forall n \in \mathbb{N}: \frac{n+1}{n} > 1$

(ii) Nechť $u \in \mathbb{R}$, $u > 1$.

chci $\exists n \in \mathbb{N}: 1 + \frac{1}{n} < u$

$\frac{1}{n} < \underbrace{u-1}_{>0}$, tj: $n > \frac{1}{u-1}$

z Arch. vl.

$$F = \left\{ \frac{m}{n},\ m, n \in \mathbb{N},\ m < n \right\}$$

0 ✓

$$\frac{m}{n} < 1$$

chci $\frac{m}{n}$ co nejmenší ... vezmu $m = 1$

$\frac{1}{n}$ ... „libovolně malé" & $n \geq 2$

$\inf F = 0$ pokud $u \in \mathbb{R}, u > 0$ $\exists n \in \mathbb{N} : n > \frac{1}{u}$, tj.

min nemá

$$F \ni \frac{1}{n} < u$$

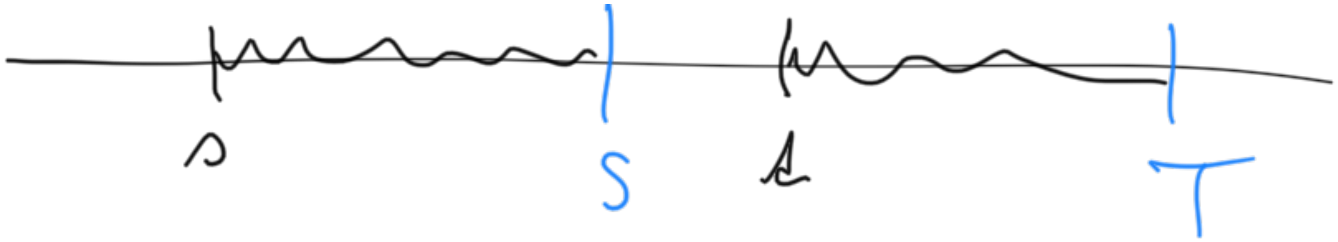
položím $n = m+1$

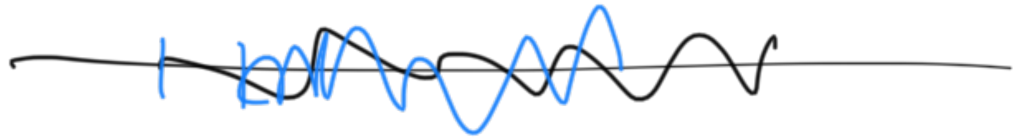
$$\frac{m}{m+1} = 1 - \frac{1}{m+1}$$

$\sup F = 1$, důkaz viz předchozí úloha (podobné)

$u < 1 - \frac{1}{m+1}$ $\quad 1$

A B B A





$\inf(A\cup B) = \min\{s,t\}$

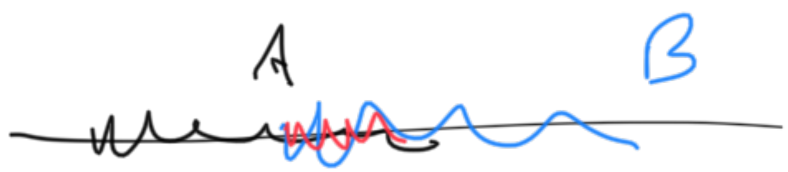
(i) $\forall x \in A\cup B$:
- $x\in A \Rightarrow x \geq s \Rightarrow x \geq \min\{s,t\}$
- $x \in B \Rightarrow x \geq t \Rightarrow x \geq \min\{s,t\}$

(ii) nechť $u \in \mathbb{R}$, $u > \min\{s,t\}$

$$\left.\begin{array}{l} \min\{s,t\}=s \ \dots\ u>s \Rightarrow \exists x\in A : x<u \\ \min\{s,t\}=t \ \dots\ u>t \Rightarrow \exists x \in B : x<u \end{array}\right\} x\in A\cup B$$

$\sup(A\cup B) = \max\{S,T\}$

Pro průnik musím předpokládat, že $A\cap B \neq \emptyset$.



$A=(-1,1)$ $\quad \inf \overset{s}{A} = -1, \ \overset{S}{\sup A} = 1$ $\quad \min\{S,T\}=1$

$B=\{-1,0,1\}$ $\quad \underset{t}{\inf B} = -1, \ \underset{T}{\sup B} = 1$ $\quad \max\{s,t\}=-1$

$A\cap B=\{0\}$ $\quad \inf A\cap B = \sup A\cap B = \underline{0}$

Platí jen $\max\{s,t\} \leq \inf(A\cap B) \leq \sup(A\cap B) \leq \min\{S,T\}$,

více říci nelze.

$\max\{s,t\}$ je dolní závora $A\cap B$

$$\Leftrightarrow \forall x \in A\cap B: \left.\begin{array}{l} x\in A \Rightarrow x\geq s \\ \& \ x\in B \Rightarrow x \geq t \end{array}\right\} \Rightarrow x \geq \max\{s,t\}$$

$A=(-1,1)$, $B=\{-1,0,\frac{1}{2}\}$