$$\lim_{x\to 0\pm} \frac{\sqrt{e^{x^6}-1}}{x\cdot\log\cos x} = \lim_{x\to 0\pm} \boxed{\frac{\cos x - 1}{\log\cos x}} \cdot \frac{\sqrt{e^{x^6}-1}}{x(\cos x-1)} \overset{AL}{\underset{\cancel{?}}{=}} 1\cdot \lim_{x\to 0\pm}\frac{\sqrt{e^{x^6}-1}}{x(\cos x -1)} =$$

↓ VOLSF

1

$f(y)=\frac{y-1}{\log y}$, $\lim_{y\to 1} f(y) \overset{AL}{=} 1$ | (P) $\cos x \neq 1$, $x\in P(0,2\pi)$

$g(x)=\cos x$, $\lim_{x\to 0} g(x) = 1$

$$= \lim_{x\to 0\pm}\frac{\sqrt{e^{x^6}-1}}{x\cdot\boxed{\frac{\cos x-1}{x^2}}\cdot x^2} \overset{AL}{\underset{\cancel{?}}{=}} -2\cdot\lim_{x\to 0\pm}\frac{\sqrt{e^{x^6}-1}}{x^3} = -2\cdot\lim_{x\to 0\pm} - \sqrt{\boxed{\frac{e^{x^6}-1}{x^6}}} =$$

↓ AL

$-\frac{1}{2}$

! $\sqrt{x^6} = |x^3|$

↓ VOLSF vnější $\frac{e^y-1}{y}$

1 vnitřní x^6

(P) $x^6\neq 0$ pro $x\neq 0$

$$= \pm 2\cdot\sqrt{1} = \pm 2$$, tedy $\lim_{x\to 0}\dots$ neexistuje

↑ VOLSF vnější $y\mapsto\sqrt{y}$

vnitřní $\frac{e^{x^6}-1}{x^6}$

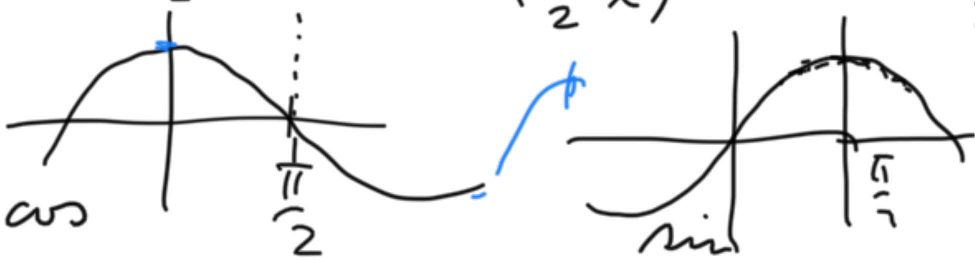
(S) ($\sqrt{}$ je spojitá v 1)

$\lim (\log\sin x)\dots^2 = \lim(\log\sin x)\cdot \sin^2 x \to 1 \quad AL \quad \log(\sin x)$

$$\lim_{x\to\frac{\pi}{2}} (\log \sin x)\cdot \operatorname{tg}^2 x = \lim_{x\to\frac{\pi}{2}} (\log \sin x)\cdot\frac{\sin^2 x}{\cos^2 x} \overset{?}{=} 1\cdot\lim_{x\to\frac{\pi}{2}} \frac{\log \sin x}{\cos^2 x} =$$

AL $\to 1$, $\frac{\sin y}{y}\to 1$, afinní subst.

$$= \lim_{x\to\frac{\pi}{2}} \frac{\log\left(\cos\left(\frac{\pi}{2}-x\right)\right)}{\sin^2\left(\frac{\pi}{2}-x\right)} = \lim_{x\to\frac{\pi}{2}} \boxed{\frac{\left(\frac{\pi}{2}-x\right)^2}{\sin^2\left(\frac{\pi}{2}-x\right)}}\cdot\frac{1}{\left(\frac{\pi}{2}-x\right)^2}\cdot\boxed{\frac{\log\left(\cos\left(\frac{\pi}{2}-x\right)\right)}{\cos\left(\frac{\pi}{2}-x\right)-1}}\cdot\left(\cos\left(\frac{\pi}{2}-x\right)-1\right)$$



$\cos x = \sin\left(\frac{\pi}{2}-x\right)$

$\sin x = \cos\left(\frac{\pi}{2}-x\right)$

$\downarrow$ VOLSF

1 $\quad f(y)=\frac{\log y}{y-1}$

$g(x)=\cos\left(\frac{\pi}{2}-x\right)\neq 1$

(P) $\quad P\left(\frac{\pi}{2}, 2\pi\right)$

$$\overset{AL}{\underset{?}{=}} 1\cdot 1\cdot \lim_{x\to\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-x\right)-1}{\left(\frac{\pi}{2}-x\right)^2} = -\frac{1}{2}$$

$\downarrow$ VOLSF

afinní subst.

$y=\frac{\pi}{2}-x$

$$\lim_{x\to 0+} \frac{e^x-e^{\sqrt{x}}}{1-\cos\sqrt{x}}\cdot\sqrt{x} = \lim_{x\to 0+} \boxed{\frac{(\sqrt{x})^2}{1-\cos\sqrt{x}}}\cdot\frac{\sqrt{x}}{(\sqrt{x})^2}\cdot\left(e^x-e^{\sqrt{x}}\right) \overset{AL}{\underset{?}{=}}$$

$\downarrow$ AL

2 VOLSF

vnitřní $\sqrt{x}$ rostoucí

(P)

$$= 2\cdot\lim_{x\to 0+}\frac{e^x-e^{\sqrt{x}}}{\sqrt{x}} = 2\cdot\lim_{x\to 0+}\left(\frac{e^x-1}{\sqrt{\ }}\cdot\frac{\sqrt{x}}{\sqrt{\ }} + \frac{1-e^{\sqrt{x}}}{\ }\right) =$$

$$\overset{AL}{=} 2\cdot\left(\lim_{x\to 0+} \underbrace{\frac{e^x-1}{x}}_{\downarrow\, 1}\cdot\underbrace{\sqrt{x}}_{\downarrow\, 0} + \underbrace{\lim_{x\to 0+}\frac{1-e^{\sqrt{x}}}{\sqrt{x}}}_{= -1}\right) \overset{AL}{=} 2\cdot(1\cdot 0 - 1) = -2$$

AL
VOLSF
vnitřní $\sqrt{x}$ rostoucí
(P)

$$\lim_{x\to\frac{3}{2}\pi} \underbrace{(4x^2-9\pi^2)\cdot\frac{\cos x}{1+\sin x}}_{f(g(x))} = \lim_{y\to 0} \overbrace{\left(4\left(y+\tfrac{3}{2}\pi\right)^2-9\pi^2\right)\cdot\frac{\cos\left(y+\frac{3}{2}\pi\right)}{1+\sin\left(y+\frac{3}{2}\pi\right)}}^{f(y)} =$$

„substituce $y = x-\frac{3}{2}\pi$"

$x = y+\frac{3}{2}\pi$

„$x\to\frac{3}{2}\pi \Rightarrow y\to 0$"

VOLSF vnější f

vnitřní $g(x) = x-\frac{3}{2}\pi$, $\lim_{x\to\frac{3}{2}\pi} g(x) = 0$

„afinní subst." g je rostoucí
(P)

$$= \lim_{y\to 0}\left(4y^2+12\pi y\right)\cdot\frac{\cos y\cdot\overbrace{\cos\tfrac{3}{2}\pi}^{0} - \sin y\cdot\overbrace{\sin\tfrac{3}{2}\pi}^{-1}}{1+\sin y\underbrace{\cos\tfrac{3}{2}\pi}_{0}+\cos y\underbrace{\sin\tfrac{3}{2}\pi}_{-1}} =$$

$$= \lim\, y(4y+12\pi)\ \frac{\sin y}{\ldots}$$

$$= \lim_{y\to 0} y(y + \dots)\cdot \frac{\quad}{1-\cos y} = \lim_{y\to 0} (4y+12\pi)\cdot\frac{\sin y}{y}\cdot\frac{y^2}{1-\cos y} \overset{AL}{=}$$

$$= 12\pi \cdot 1 \cdot 2 = 24\pi$$

$$\lim_{n\to\infty} \underbrace{\left(\log(n^6+5n^3+1) - \log(n^6+1)\right)\cdot(n^3+\cos n)}_{f(x_n)} \overset{HEINE}{=} \cancel{?}$$

$$f(x) = \left(\log(x^6+5x^3+1) - \log(x^6+1)\right)\cdot(x^3+\cos x)$$

$$x_n = n \ne +\infty = c$$

Heineova věta

$$\lim_{n\to\infty} f(x_n) = \lim_{x\to c} f(x) = \lim_{x\to+\infty} f(x)$$, pokud existuje

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} n = +\infty$$

$$\Rightarrow c = +\infty$$

$$= \lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \left(\log \overbrace{\frac{x^6+5x^3+1}{x^6+1}}^{\to 1}\right)\cdot(x^3+\cos x) =$$

$$= \lim_{x\to\dots} \boxed{\frac{\log\frac{x^6+5x^3+1}{x^6+1}}{\dots}} \cdot \frac{\cancel{x^6}+5x^3+\cancel{1}-\cancel{x^6}-\cancel{1}}{\dots}\cdot(x^3+\cos x) =$$

$$\lim_{x\to+\infty} \boxed{\frac{x^6+5x^3+1}{x^6+1} - 1} \qquad x^6+1$$

VOLSF vnější: $\frac{\log_3}{0 \to 1}$ $\to 1$

vnitřní: $\frac{x^6+5x^3+1}{x^6+1} \neq 1$ pro $x \neq 0 \Rightarrow (P)$ „$\lim_{x\to+\infty} g(x)$"

$\cancel{x^6}+5x^3+\cancel{1} \neq \cancel{x^6}+\cancel{1}$

$5x^3 \neq 0$

$$\overset{AL}{=} 1 \cdot \lim_{x\to+\infty} \frac{5x^6+5x^3\cos x}{x^6+1} = \lim_{x\to+\infty} \frac{5+\frac{5}{x^3}\cdot\cos x}{1+\frac{1}{x^6}} \overset{AL}{=} \frac{5+0}{1+0} = 5$$

$\frac{5}{x^3}\cdot\cos x \to 0$ „nulová x omezená"

$\frac{1}{x^6} \to 0$