$$f(x)=\sqrt{e^{\sin^2 x}-1}$$

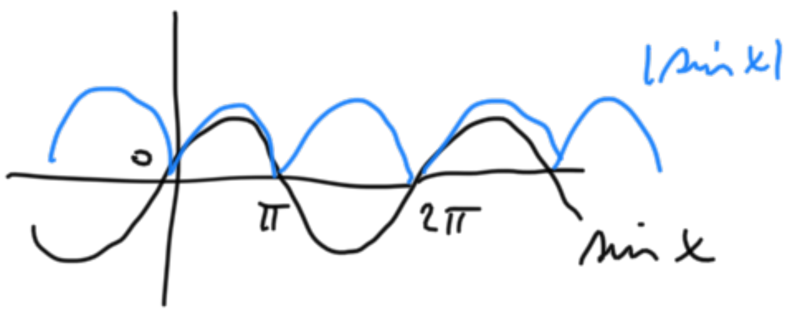
$e^{\sin^2 x}\geq 1$ $\quad D_f=\mathbb{R}$, f spojitá na $\mathbb{R}$ (aritmetika + skládání spoj. fcí)

$\sin^2 x\geq 0$

vždy

f je 2π-periodická, dokonce π-periodická

$\sin^2 x=|\sin x|^2$ $\qquad$ stačí vyšetřovat např. na $\langle 0,\pi\rangle$



$$f'(x)=\frac{1}{2\sqrt{e^{\sin^2 x}-1}}\cdot e^{\sin^2 x}\cdot 2\sin x\cdot\cos x=$$

$$=\frac{e^{\sin^2 x}\cdot\sin x\cdot\cos x}{\sqrt{e^{\sin^2 x}-1}},\quad x\in(0,\pi)$$

$x\in(k\pi,\pi+k\pi),\ k\in\mathbb{Z}$

$$e^{\sin^2 x}-1=0$$
$$\sin^2 x=0$$
$$\sin x=0$$
$$x=k\pi,\ k\in\mathbb{Z}$$

$f'_+(0)=$ < z definice

$\lim f'(x)$ (pokud ...)

$\lim\limits_{x\to 0+} f(x)$ [illegible]

z def.

$$= \lim_{x\to 0+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0+} \frac{\sqrt{e^{\sin^2 x}-1}}{x} = \lim_{x\to 0+} \sqrt{\frac{e^{\sin^2 x}-1}{x^2}} =$$

$$= \lim_{x\to 0+} \sqrt{\underbrace{\frac{e^{\sin^2 x}-1}{\sin^2 x}}_{\downarrow \atop 1} \cdot \underbrace{\frac{\sin^2 x}{x^2}}_{\downarrow \atop 1}} = 1$$

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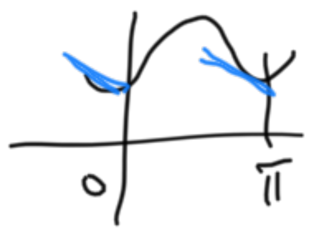
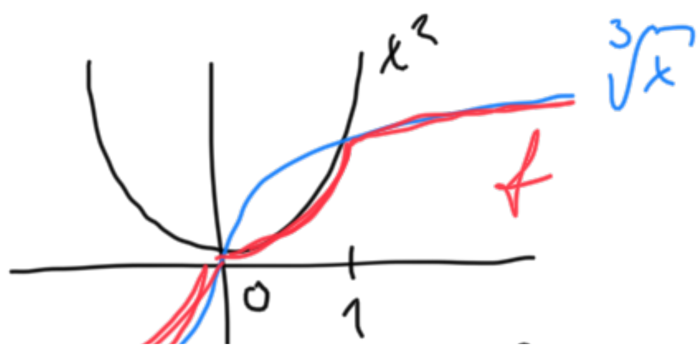
$$f'_-(\pi) = f'_-(0) = \lim_{x\to 0-} \frac{\sqrt{e^{\sin^2 x}-1}}{x} = -\lim_{x\to 0-} \sqrt{\frac{e^{\sin^2 x}-1}{x^2}} = -1$$



$\Rightarrow f'(0)$ neexistuje
$f'(\pi)$ neexistuje

$$f(x) = \min\{x^2, \sqrt[3]{x}\}$$

$D_f = \mathbb{R}$, f je spojitá na $\mathbb{R}$ (min. ze spoj. fcí)



$$f(x) = \begin{cases} \sqrt[3]{x}, & x \in (-\infty, 0\rangle \cup \langle 1, +\infty) \\ x^2, & x \in \langle 0, 1\rangle \end{cases}$$

$$x^2 = \sqrt[3]{x}$$
$$x^6 = x$$
$$x^5 = 1$$

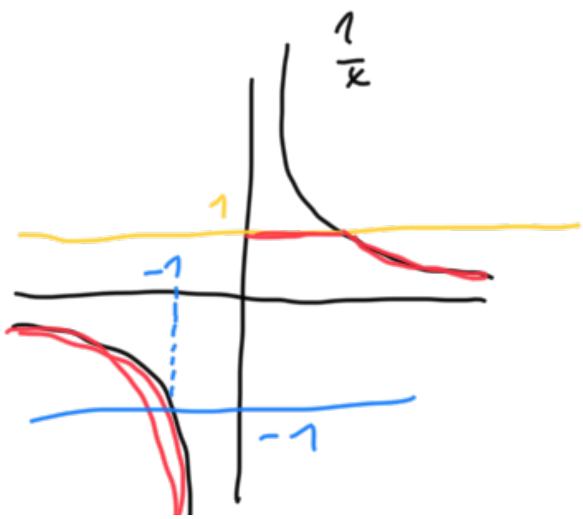
f je spojitá

$$f'(x) = \begin{cases} \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}, & x \in (-\infty, 0) \cup (1, +\infty) \\ 2x, & x \in (0, 1) \end{cases}$$

$$\left.\begin{aligned} f'_-(0) &= \lim_{x \to 0-} f'(x) = \lim_{x \to 0-} \frac{1}{3} \frac{1}{\sqrt[3]{x^2}} = +\infty \quad \left(\text{„}\frac{1}{\text{kladná } 0}\text{“}\right) \\ f'_+(0) &= \lim_{x \to 0+} f'(x) = \lim_{x \to 0+} 2x = 0 \end{aligned}\right\} f'(0) \text{ neexistuje}$$

$$\left.\begin{aligned} f'_-(1) &= \lim_{x \to 1-} f'(x) = \lim_{x \to 1-} 2x = 2 \\ f'_+(1) &= \lim_{x \to 1+} f'(x) = \lim_{x \to 1+} \frac{1}{3} \frac{1}{\sqrt[3]{x^2}} = \frac{1}{3} \end{aligned}\right\} f'(1) \text{ neexistuje}$$

$$f(x) = \arcsin \underbrace{\min\left\{1, \frac{1}{x}\right\}}_{g}$$



$$D_g = \mathbb{R} \setminus \{0\}$$

$$g(x) = \begin{cases} 1, & x \in (0, 1\rangle \\ \frac{1}{x}, & x \in (-\infty, 0) \cup \langle 1, +\infty) \end{cases}$$

g je spojitá na D_g

D_f: $-1 \leq g(x) \leq 1$

$D_f = (-\infty, -1\rangle \cup (0, +\infty)$

$$f(x) = \begin{cases} \arcsin 1 = \frac{\pi}{2}, & x \in (0,1\rangle \\ \arcsin \frac{1}{x}, & x \in (-\infty,-1\rangle \cup \langle 1,+\infty) \end{cases}$$

f je spojitá na D_f (skládání spoj. fcí)

$$f'(x) = \begin{cases} 0, & x \in (0,1) \\ \frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right) = -\frac{1}{\sqrt{\frac{x^2-1}{x^2}} \cdot x^2} = -\frac{1}{\frac{\sqrt{x^2-1}}{|x|} \cdot x^2} = -\frac{1}{|x| \cdot \sqrt{x^2-1}}, & \end{cases}$$

$\underline{x \in (-\infty,-1) \cup (1,+\infty)}$

f spojitá

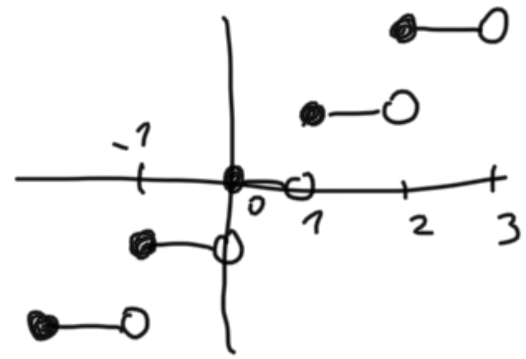
$$f'_-(-1) \overset{\Downarrow}{=} \lim_{x \to -1-} f'(x) = \lim_{x \to -1-} -\frac{1}{|x|\sqrt{x^2-1}} =$$

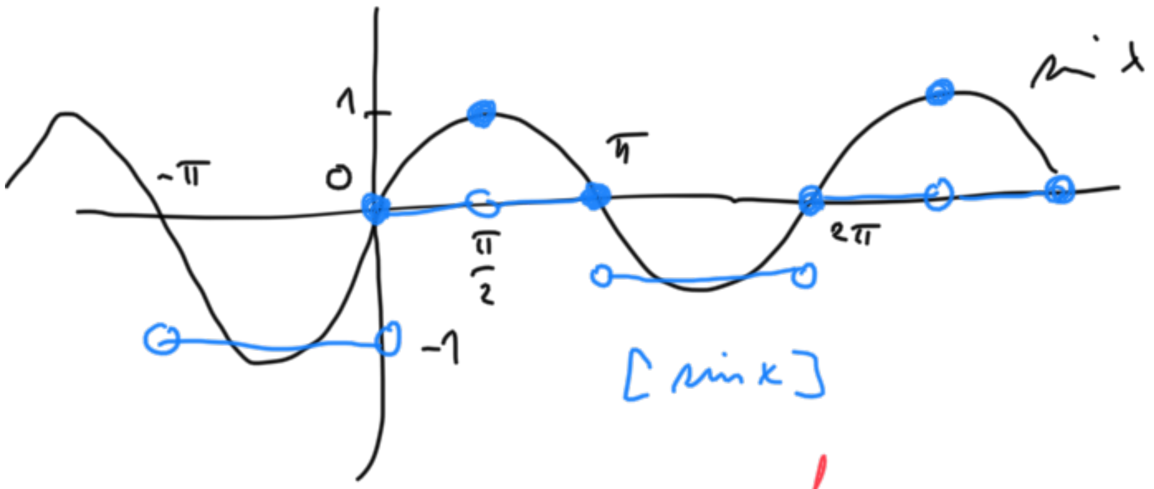
$$= -\infty$$

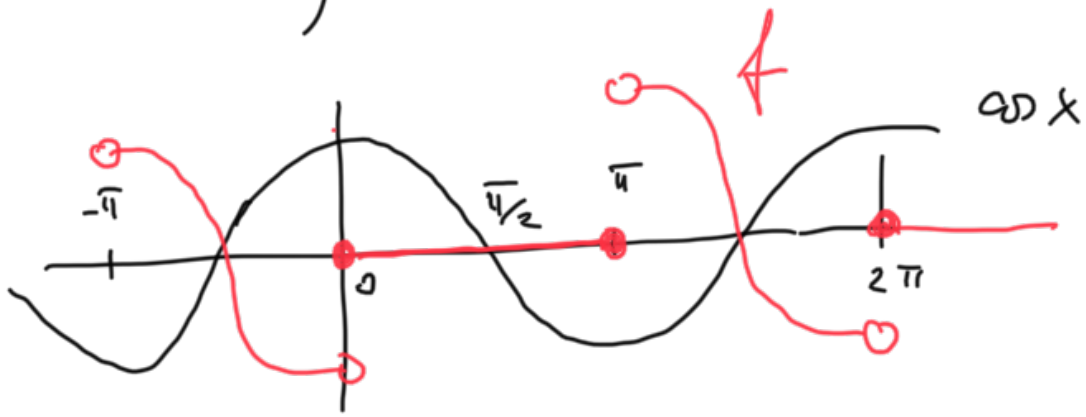
$$\left.\begin{aligned} f'_-(1) &= \lim_{x \to 1-} 0 = 0 \\ f'_+(1) &= \lim_{x \to 1+} -\frac{1}{|x|\sqrt{x^2-1}} = -\infty \end{aligned}\right\} f'(1) \text{ neexistuje}$$

$f(x) = \cos x \cdot [\sin x]$

↑ celá část







$$f(x) = \begin{cases} 0, & x \in \langle 0, \pi\rangle + 2k\pi, \ k \in \mathbb{Z} \\ -\cos x, & x \in (\pi, 2\pi) + 2k\pi, \ k \in \mathbb{Z} \end{cases}$$

↑ f spojitá v 0 zprava, π zleva

f spojitá na $\langle 0, \pi\rangle + 2k\pi$

— " — $(\pi, 2\pi) + 2k\pi$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} -\cos x = -1 \neq 0 = f(0)$$

f není spojitá v 0 zleva

$$\lim_{x \to \pi+} f(x) = \lim_{x \to \pi+} -\cos x = 1 \neq 0 = f(\pi)$$

f není spojitá v π zprava

(v 2π podobně jako v 0, neboť f je 2π-periodická) (v 2π stejné jako v 0 ...

$$f'(x) = \begin{cases} 0, & x \in (0,\pi) + 2k\pi,\ k \in \mathbb{Z} \\ \sin x, & x \in (\pi, 2\pi) + 2k\pi,\ k \in \mathbb{Z} \end{cases}$$

$$f'_+(0) = \lim_{x \to 0+} f'(x) = \lim_{x \to 0+} 0 = 0$$

↑ f spojitá v 0 zprava

$$f'_-(\pi) = \lim_{x \to \pi-} f'(x) = 0$$

↑ f spojitá v π zleva

$$f'_-(0) \overset{\text{def.}}{=} \lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0-} \frac{-\cos x}{x} = -(-\infty) = +\infty$$

$$f'_+(\pi) \overset{\text{def.}}{=} \lim_{x \to \pi+} \frac{-\cos x}{x - \pi} = +\infty$$

↑ "kladná / 0"

$f'(0)$, $f'(\pi)$ neexistují

$$f(x) = (\operatorname{sgn} x) \cdot g(x)$$

$$f(x) = \begin{cases} g(x), & x \in (0,+\infty) \\ -g(x), & x \in (-\infty,0) \\ 0, & x=0 \end{cases}$$

~~$f'(0)=0$, protože $f'(0)=0$, což je konstanta~~ !

$f(x) = x$ $\qquad$ $f(x) = \begin{cases} x, & x \in \mathbb{R} \setminus \{0\} \\ 0, & x=0 \end{cases}$

$$f(x) = \log\left(x + \frac{1}{x}\right)$$

$x \neq 0$ $\quad$ $x + \frac{1}{x} > 0$
$x<0$ neplatí
$x>0$ platí

$\Bigg\}$ $D_f = (0,+\infty)$, f spojitá na D_f (aritmetika + skládání spoj. fcí)

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} \log\underbrace{\left(x+\frac{1}{x}\right)}_{\downarrow\, +\infty} = +\infty$$

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$$\lim_{x\to +\infty} \log\left(\underbrace{x+\frac{1}{x}}\right) = +\infty$$

$\Downarrow$
$+\infty$

$$f'(x) = \frac{1}{x+\frac{1}{x}} \cdot \left(1 - \frac{1}{x^2}\right) = \frac{1}{\frac{x^2+1}{x}} \cdot \frac{x^2-1}{x^2} = \frac{(x+1)(x-1)}{x(x^2+1)}, \quad \underline{x \in (0,+\infty)}$$

$x(x^2+1)$ "
x^3+x

$$\begin{array}{ccc} & - & + \\ \hline & 0 & 1 \end{array}$$

x	$(0,1)$	$(1,+\infty)$
f'	<0	>0
f	klesající	rostoucí

„hrubý průběh"

$f(1) = \log 2 > 0$

f má v 1 glob. minimum

jiné glob. ani lokální extrémy nemá

$$H_f = \langle \log 2, +\infty)$$

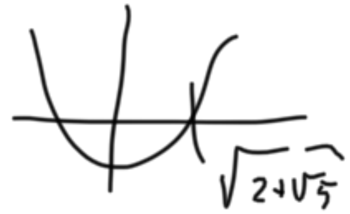
$$f''(x) = \frac{2x \cdot x(x^2+1) - (x+1)(x-1)\cdot(3x^2+1)}{x^2(x^2+1)^2} = \frac{2x^4+2x^2-(x^2-1)(3x^2+1)}{x^2(x^2+1)^2} =$$

$$2x^4+2x^2-3x^4+3x^2-x^2+1 \quad = \quad -x^4+4x^2+1 \quad = \quad x^4-4x^2-1$$

$$= \frac{\qquad}{x^2(x^2+1)^2} = \frac{\qquad}{x^2(x^2+1)^2} - - \frac{\qquad}{x^2(x^2+1)^2}$$

$x \in (0, +\infty)$

$$x^4 - 4x^2 - 1 > 0$$



$$D = 16 + 4 = 20$$

$y = x^2$ $\qquad y_{1,2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$$x^2 = 2 + \sqrt{5}$$

$$x = \sqrt{2+\sqrt{5}}$$

x	$(0, \sqrt{2+\sqrt{5}})$	$(\sqrt{2+\sqrt{5}}, +\infty)$
f''	> 0	< 0
f	ryze konvexní	ryze konkávní

bod $\sqrt{2+\sqrt{5}}$ je inflexní bod

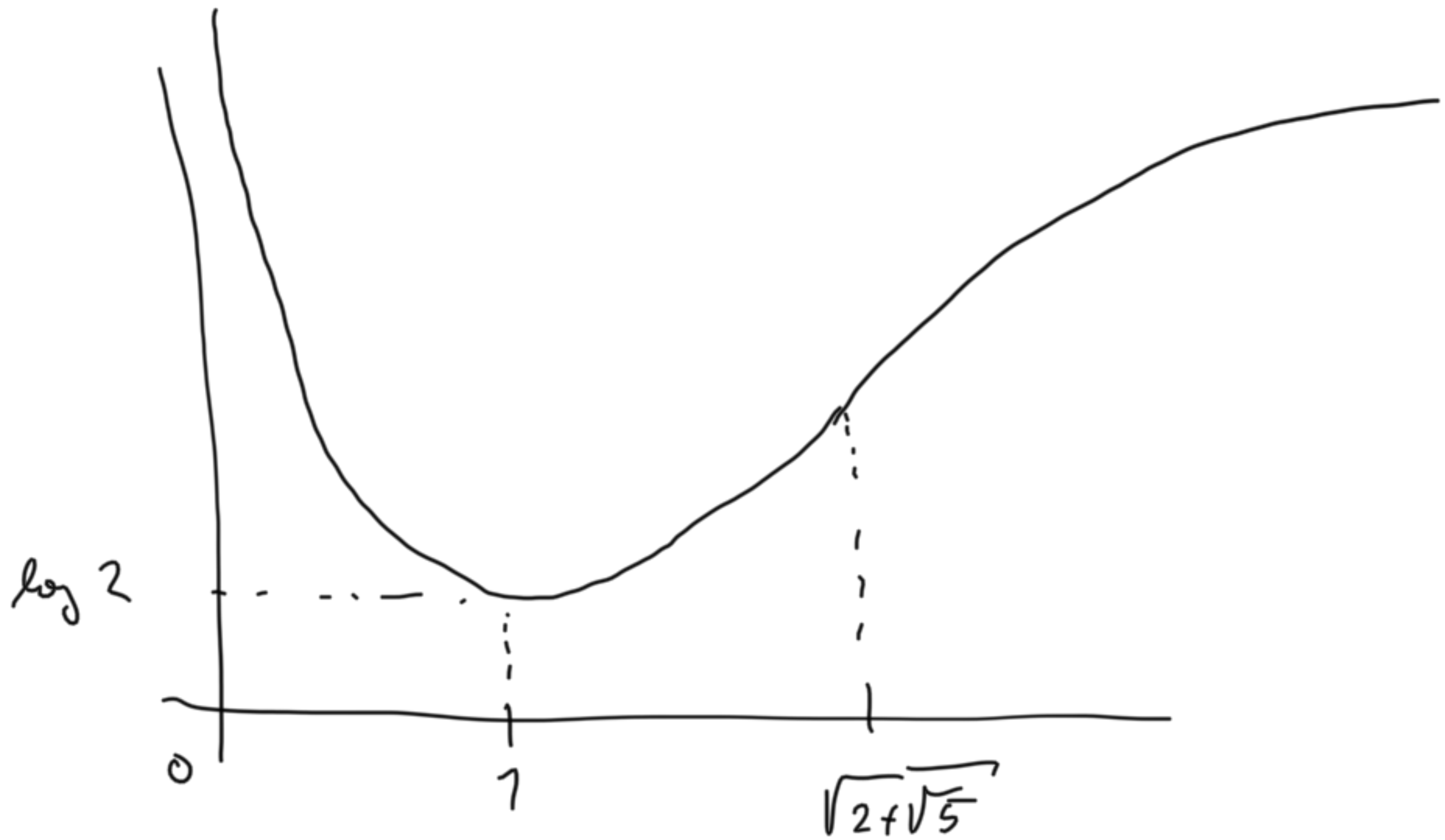
asymptota v $+\infty$: $\lim\limits_{x \to +\infty} \frac{\log\left(x + \frac{1}{x}\right)}{x} \underset{\text{L'H, typ } \frac{+\infty}{+\infty}}{=} \lim\limits_{x \to +\infty} \frac{\frac{(x+1)(x-1)}{x(x^2+1)}}{1} = \ldots = 0$

2POL: $\frac{\log x}{x} \leq \frac{\log\left(x + \frac{1}{x}\right)}{x} \leq \frac{\log 2x}{x}$

$\downarrow \qquad\qquad \downarrow$ R.Š.

$$\lim_{x\to+\infty}(f(x)-x) = \lim_{x\to+\infty}\left(\log\left(x+\frac{1}{x}\right)-0\cdot x\right)=+\infty \Rightarrow \text{asymptota v } +\infty$$

neexistuje



$$f(x)=\sqrt[5]{3x^5+5x^3} \qquad D_f = R\,,\ f \text{ je spojitá na } R \text{ (skl. spojitých)}$$

$$f(-x)=\sqrt[5]{-3x^5-5x^3} = -\sqrt[5]{3x^5+5x^3} = -f(x) \ \ldots\ldots\ f \text{ je lichá}$$

stačí vyšetřovat na $\langle 0,+\infty)$

$$\lim_{x\to+\infty} f(x) = +\infty\,,\ \lim_{x\to-\infty} f(x) = -\infty$$

$$f'(x) = \frac{1}{5\sqrt[5]{(3x^5+5x^3)^4}} \cdot (15x^4+15x^2) = \frac{3x^2(x^2+1)}{\sqrt[5]{x^{12}(3x^2+5)^4}} =$$

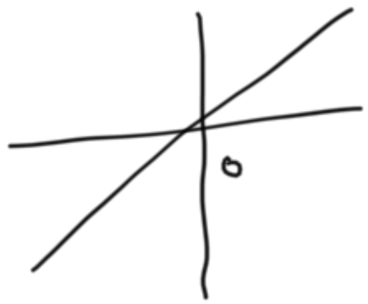
$$= \frac{3(x^2+1)}{\sqrt[5]{x^2}\cdot\sqrt[5]{(3x^2+5)^4}} \quad \Big| \quad \underline{x \in \mathbb{R}\setminus\{0\}}$$

$$3x^5+5x^3=0$$
$$x^3(3x^2+5)=0$$
$$x=0$$

$$f'(0) \underset{\substack{\uparrow \\ f \text{ spoj. v } 0}}{=} \lim_{x\to 0} f'(x) = \lim_{x\to 0} \frac{3(x^2+1)}{\sqrt[5]{x^2}\cdot\sqrt[5]{(3x^2+5)^4}} = +\infty$$

$f'(x) > 0$ pro $\forall x \in \mathbb{R}$ $\Rightarrow$ f je rostoucí na $\mathbb{R}$

$f(0)=0$



f nemá glob. ani lokální extrémy

$H_f = \mathbb{R}$

$$f''(x) = \frac{6x\cdot\sqrt[5]{x^2(3x^2+5)^4} - 3(x^2+1)\,\dfrac{1}{5\sqrt[5]{(x^2(3x^2+5)^4)^4}}\cdot\left(2x(3x^2+5)^4 + x^2\,4(3x^2+5)^3\cdot 6x\right)}{\sqrt[5]{x^4(3x^2+5)^8}} =$$

$6x^2+10+24x^2 = 30x^2+10$

$$= \frac{5 \cdot 6x \cdot \left(x^2 \cdot (3x^2+5)^4\right) - 3(x^2+1)\left(x(3x^2+5)^3\left(2(3x^2+5) + 24x^2\right)\right)}{\sqrt[5]{x^4(3x^2+5)^8} \cdot \sqrt[5]{\left(x^2(3x^2+5)^4\right)^4} \cdot 5} =$$

$$= \frac{30x^3(3x^2+5)^4 - 3(x^2+1) \cdot x(3x^2+5)^3 \cdot 10(3x^2+1)}{5\sqrt[5]{x^{12}(3x^2+5)^{24}}} =$$

$$= \frac{6(3x^2+5)^3\left(x^3(3x^2+5) - x(x^2+1)(3x^2+1)\right)}{\sqrt[5]{x^{12}(3x^2+5)^{24}}} =$$

$$= \frac{6x(3x^2+5)^3\left(\overbrace{3x^4+5x^2-3x^4-x^2-3x^2-1}^{x^2-1}\right)}{} =$$

$$= \frac{6(x^2-1)}{\sqrt[5]{x^7 \cdot (3x^2+5)^9}}, \quad x \in \mathbb{R} \setminus \{0\}$$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -1 & & 0 & & 1 & \end{array}$$

x	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,+\infty)$
y''	<0	>0	<0	>0

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
f	ryze konkávní	r. konvexní	r. konkávní	r. konvexní

Body -1 a 1 jsou inflexní body.

Bod 0 <u>není</u> inflexní bod (v 0 není tečna, neboť $f'(0)=+\infty$).

asymptota v $+\infty$: $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to+\infty}\sqrt[5]{\dfrac{3x^5+5x^3}{x^5}}=\sqrt[5]{3}$

$$\lim_{x\to+\infty}\left(f(x)-\sqrt[5]{3}\,x\right)=\lim_{x\to+\infty}\frac{\cancel{3x^5}+5x^3-\cancel{3x^5}}{\left(\sqrt[5]{3x^5+5x^3}\right)^4+\dots}=\dots=0$$

$\to\ y=\sqrt[5]{3}\,x$

asymptota v $-\infty$ stejná