$$f(x) = \sqrt[5]{1-\sqrt{x+1}}$$

$x+1 \geq 0$

$x \geq -1$

$$D_f = \langle -1, +\infty)$$

$f$ je spojitá na $D_f$ (aritmetika + skládání spoj. fcí)

$$f(-1) = 1$$

$$\lim_{x \to +\infty} f(x) = -\infty$$

$$f'(x) = \frac{1}{5} \cdot \frac{1}{\sqrt[5]{(1-\sqrt{x+1})^4}} \cdot \frac{-1}{2\sqrt{x+1}} = -\frac{1}{10} \cdot \frac{1}{\sqrt{x+1}} \cdot \frac{1}{\sqrt[5]{(1-\sqrt{x+1})^4}}$$

$\underline{x \in (-1,0) \cup (0,+\infty)}$

$$f'_+(-1) = \underset{\substack{\uparrow \\ \text{spoj. } f}}{\lim_{x \to -1_+}} f'(x) = -\infty$$

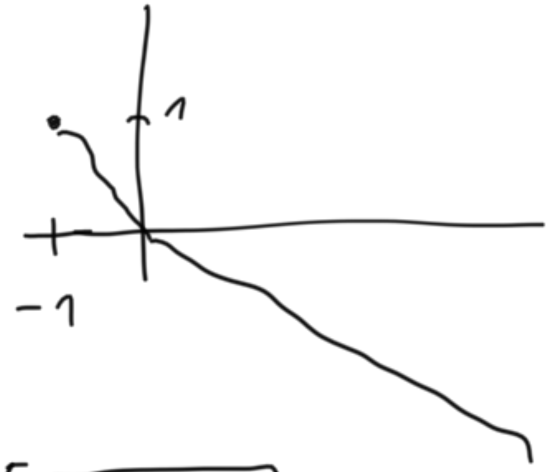
$f' < 0$ na $(-1, +\infty)$

$\Rightarrow f$ je klesající na $D_f = \langle -1, +\infty)$

[illegible]

$$f(0) = \lim_{x\to 0} f(x) = -\infty$$

$f(0)=0$



glob. max. je v $-1$

jiné glob. ani lok. nejsou

$H_f = (-\infty, 1\rangle$

$$f''(x) = -\frac{1}{10}\cdot\frac{-1\cdot\left(\frac{1}{2\sqrt{x+1}}\cdot\sqrt[5]{(1-\sqrt{x+1})^4} + \sqrt{x+1}\cdot\frac{4}{5}\frac{1}{\sqrt[5]{1-\sqrt{x+1}}}\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{\sqrt{x+1}}\right)}{(x+1)\sqrt[5]{(1-\sqrt{x+1})^8}} =$$

$$= \frac{1}{10}\,\frac{\frac{1}{2}\left(1-\sqrt{x+1}\right) - \frac{4}{10}\sqrt{x+1}}{(x+1)\sqrt{x+1}\cdot\sqrt[5]{(1-\sqrt{x+1})^9}} = \frac{1}{100}\cdot\frac{5-9\sqrt{x+1}}{\underset{\vee}{(x+1)}\,\underset{\vee}{\sqrt{x+1}}\cdot\underbrace{\sqrt[5]{(1-\sqrt{x+1})^9}}}$$

$\phantom{x}\qquad 0 \qquad 0$

$x \in (-1,0) \cup (0,+\infty)$

| | |
|---|---|
| $1-\sqrt{x+1} > 0$ | $5-9\sqrt{x+1} > 0$ |
| $\sqrt{x+1} < 1$ | $\sqrt{x+1} < \frac{5}{9}$ |
| $x+1 < 1$ | $x+1 < \frac{25}{81}$ |
| $x < 0$ | $x < -\frac{56}{81}$ |

$-1 \quad + \quad -\frac{56}{81} \quad - \quad 0 \quad +$

| $x \in$ | $(-1, -\frac{56}{81})$ | $(-\frac{56}{81}, 0)$ | $(0,+\infty)$ |
|---|---|---|---|
| $f''$ | $>0$ | $<0$ | $>0$ |

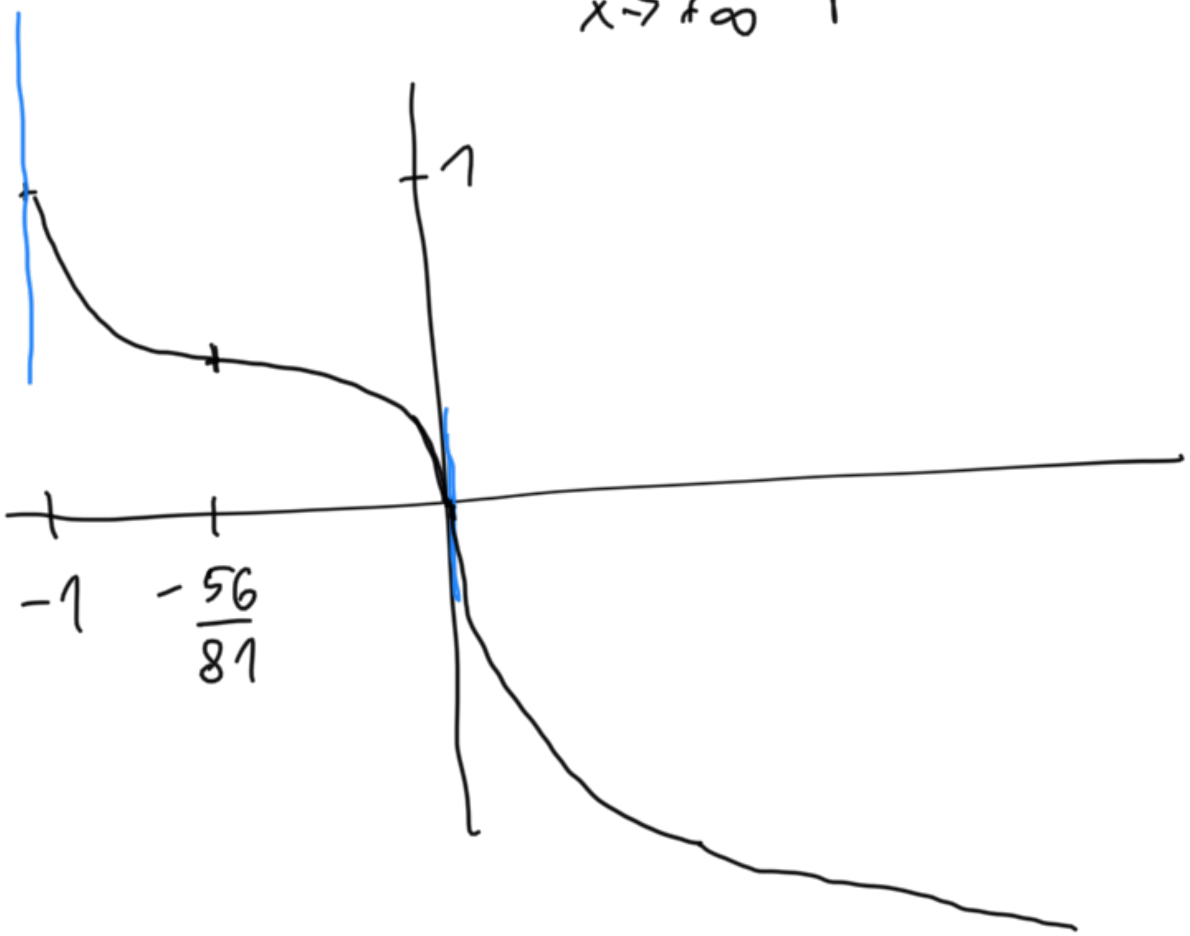
bod $-\frac{56}{81}$ je inflexní

| $f''$ | $>0$ | $<0$ | $>0$ |
|---|---|---|---|
| $f$ | ryze konvexní | r. konkávní | r. konvexní |

(bod 0 není inflexní)

asymptota v $+\infty$: $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \sqrt[5]{\frac{1-\sqrt{x+1}}{x^5}} = 0$

$\lim\limits_{x\to+\infty} f(x) - 0\cdot x = -\infty \Rightarrow$ asymptota neexistuje




$$f(x) = \sin x + \frac{1}{6\sin x}$$

$$D_f = \mathbb{R} \setminus \{k\pi;\ k \in \mathbb{Z}\} = \bigcup_{k \in \mathbb{Z}} (k\pi, \pi + k\pi)$$

$f$ je spojitá na $D_f$

$f$ je $2\pi$-periodická

$$f(-x) = \sin(-x) + \frac{1}{6\sin(-x)} = -\sin x + \frac{1}{-6\sin x} = -f(x)$$

$f$ je lichá

stačí vyšetřovat jen na $\underline{(0,\pi)}$

$$\lim_{x \to 0+} f(x) = +\infty$$

$$\lim_{x \to \pi-} f(x) = +\infty$$

$$f'(x) = \cos x + \frac{1}{6} \cdot \frac{-1}{\sin^2 x} \cdot \cos x = \cos x \left(1 - \frac{1}{6\sin^2 x}\right), \ \underline{x \in (0,\pi)}$$

$\cos x > 0$ | $1 - \frac{1}{6\sin^2 x} > 0$

$x \in (0, \pi)$

$\arcsin \frac{1}{\sqrt{6}}$ $\pi - \arcsin \frac{1}{\sqrt{6}}$

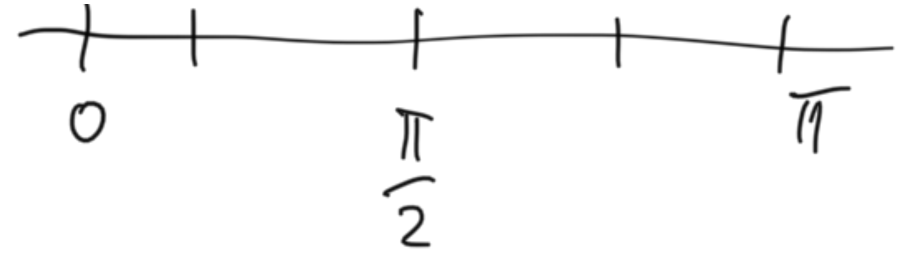
$x \in (0, \frac{\pi}{2})$

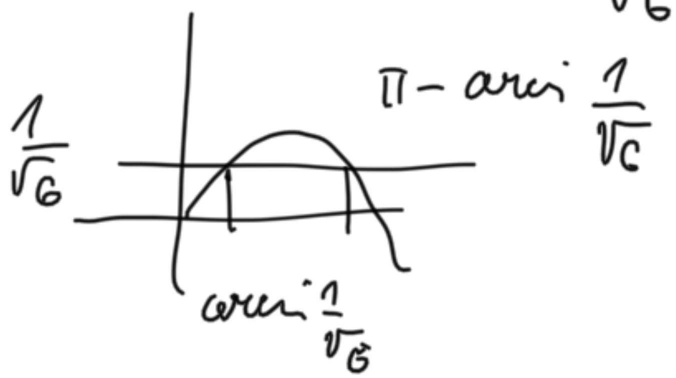
$$\frac{1}{6\sin^2 x} < 1$$

$$\sin^2 x > \frac{1}{6}$$

$\sin x > 0$ na $(0,\pi)$ $\longrightarrow$

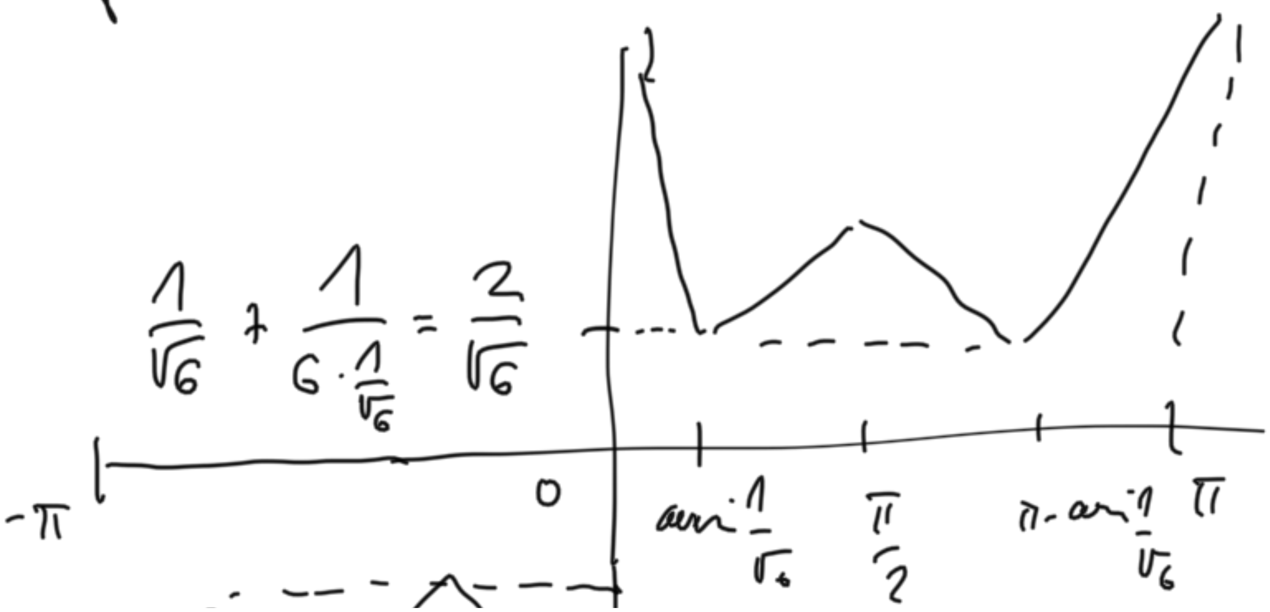
$$\sin x > \frac{1}{\sqrt{6}} \iff x \in \left(\arcsin\frac{1}{\sqrt{6}}, \pi - \arcsin\frac{1}{\sqrt{6}}\right)$$




| $x \in$ | $(0, \arcsin\frac{1}{\sqrt{6}})$ | $(\arcsin\frac{1}{\sqrt{6}}, \frac{\pi}{2})$ | $(\frac{\pi}{2}, \pi - \arcsin\frac{1}{\sqrt{6}})$ | $(\pi - \arcsin\frac{1}{\sqrt{6}}, \pi)$ |
|---|---|---|---|---|
| $f'$ | $< 0$ | $> 0$ | $< 0$ | $> 0$ |
| $f$ | klesající | rostoucí | kles. | rost. |

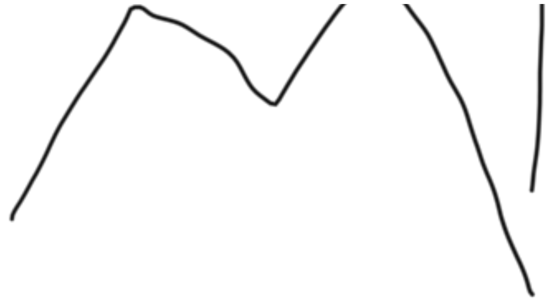
$$\frac{1}{\sqrt{6}} + \frac{1}{6 \cdot \frac{1}{\sqrt{6}}} = \frac{2}{\sqrt{6}}$$




v bodech $\arcsin\frac{1}{\sqrt{6}}$, $\pi - \arcsin\frac{1}{\sqrt{6}}$ je glob. minimum

v $\frac{\pi}{2}$ je lokální maximum

jiné extrémy nejsou



$$H_f = \left(-\infty, -\frac{2}{\sqrt{6}}\right\rangle \cup \left\langle \frac{2}{\sqrt{6}}, +\infty\right)$$

$$f''(x) = -\sin x\left(1 - \frac{1}{6\sin^2 x}\right) + \cos x \cdot \left(-\frac{1}{6}\right)(-2)\frac{1}{\sin^3 x}\cdot \cos x =$$

$$= -\sin x + \frac{1}{6\sin x} + \frac{\cos^2 x}{3\sin^3 x} = \frac{-6\sin^4 x + \sin^2 x + 2\overbrace{\cos^2 x}^{1-\sin^2 x}}{6\sin^3 x} =$$

$-6y^2 - y + 2,\ y = \sin^2 x$

$$= \frac{-6\sin^4 x + 2 - \sin^2 x}{6\sin^3 x} = \frac{\overbrace{(3\sin^2 x + 2)}^{>0}(1 - 2\sin^2 x)}{\underbrace{6\sin^3 x}_{>0}}, \quad x \in (0,\pi)$$

$1 - 2\sin^2 x > 0$

$\sin^2 x < \frac{1}{2}$

$\sin x > 0 \rightarrow x \in (0,\pi)$

$\sin x < \frac{1}{\sqrt{2}}$

$x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{3}{4}\pi, \pi\right)$

| $x \in$ | $\left(0, \frac{\pi}{4}\right)$ | $\left(\frac{\pi}{4}, \frac{3}{4}\pi\right)$ | $\left(\frac{3}{4}\pi, \pi\right)$ |
|---|---|---|---|
| $f''$ | $>0$ | $<0$ | $>0$ |
| $f$ | ryze konvexní | r. konk. | r. konvexní |

$\frac{\pi}{4}, \frac{3}{4}\pi$ punti inflessione