Let X be a Frechet space, Y a LCS
$T_n : X \to Y$ continuous linear mappings
and $Tx = \lim_{n\to\infty} T_n x$ exists for each $x \in X$

Then T is a continuous linear mapping

Proof: Clearly T is linear (note that Y is a TVS,
hence $u_n \to u$ and $v_n \to v$ in Y implies $u_n + v_n \to u+v$
and $u_n \to u$ in Y implies $\lambda u_n \to \lambda u$ for $\lambda \in \mathbb{F}$)

Let us prove that T is continuous.
Fix q, any continuous seminorm on Y

Then for each $x \in X$: $q(T_n x) \to q(Tx)$
$\Rightarrow$ sequence $(q(T_n x))$ is bounded
For $m \in \mathbb{N}$ set
$M_m = \{x \in X ;\ \forall n \in \mathbb{N} : q(T_n x) \leq m\}$
$\Rightarrow$ M_m is a closed absolutely convex set
and $X = \bigcup_m M_m$

Baire Category theorem $\Rightarrow \exists m : \operatorname{Int} M_m \neq \emptyset$
(X is metrizable by a complete metric)

$\Rightarrow \exists U$ absolutely convex open neighbourhood of 0 in X
$\exists x \in X : x + U \subset M_m$

M_m balanced, hence symmetric $\Rightarrow -(x+U) \subset M_m$
$\quad\quad -(x+U) = -x + U \quad (U = -U)$

$\Rightarrow x + U \subset M_m \quad -x + U \subset M_m$
M_m convex $\Rightarrow U \subset M_m$
Then p_U is a continuous seminorm on X
and $q(Tx) \leq m \cdot q_U(x)$
$\Big[\ q_U(x) < c \Rightarrow q_U(\frac{x}{c}) < 1 \Rightarrow \frac{x}{c} \in U \subset M_m \Rightarrow \forall n : q(T_n \frac{x}{c}) \leq m$
$\Rightarrow q(T_n x) \leq m \cdot c \Rightarrow q(Tx) \leq m \cdot c\ \Big]$ Hence T is continuous
$q(T_n x) \to q(Tx)$