Theorem XI.15 Let X be a Banach space

Set $K = (B_{X^*}, w^*)$. Then K is a compact Hausdorff space

$J : X \to C(K)$ defined by $J(x) = \hat{x}(x)\restriction_K$,

i.e. $J(x)(x^*) = x^*(x)$, $x \in K$, is a linear isometry,

$w \to \tau_p$ homeomorphism, $J(X)$ is τ_p-closed

Proof ① It is clear that J is a well-defined linear operator

② J is an isometry:

$\|J(x)\| = \sup\{|x^*(x)| ; x^* \in B_{X^*}\} = \|x\|$ dual formula for the norm

③ J is $w \to \tau_p$ cts

⌈Fix $x^* \in K = B_{X^*}$ then $J(x)(x^*) = x^*(x)$,

i.e. $(x \mapsto J(x)(x^*)) = x^*$, which is weakly cts ⌋

④ J^{-1} is $\tau_p \to w$ cts

⌈Fix $x^* \in X^*$. Find $c > 0$ s.t. $\frac{x^*}{c} \in B_{X^*}$.

Then for each $f \in J(X)$ we have

$J^{-1}(f)(x^*) = c\, J^{-1}(f)(\frac{x^*}{c}) = c \cdot f(\frac{x^*}{c})$,

so $f \mapsto J^{-1}(f)(x^*)$ is τ_p-cts ⌋

⑤ $\mathbb{F} = \mathbb{R} \Rightarrow J(X) = \{f \in C(K) ; f \text{ affine}, \& f(0) = 0\}$

$\mathbb{F} = \mathbb{C} \Rightarrow J(X) = \{f \in C(K) ; f \text{ affine}, f(0) = 0$

$\& \forall \alpha \in \mathbb{C}, |\alpha| = 1 : f(\alpha x^*) = \alpha f(x^*)$

$\forall x^* \in K \}$

⌈Recall : f affine $\Leftrightarrow \forall x^*, y^* \in K \ \forall t \in [0,1]$

$f(t x^* + (1-t) y^*) = t f(x^*) + (1-t) f(y^*)$ ⌋

"$\subseteq$" clear

"$\supseteq$" $f \in$ Right-hand side $\Rightarrow \exists\, g : X^* \to \mathbb{F}$ linear

$g \upharpoonright K = f$

$\shortparallel$

$g \upharpoonright B_{X^*}$

. By Corollary 15 $f \in \mathcal{H}(+)$,

so $g \in J(X)$

And, it is clear that the subspaces on the RHS are τ_p-closed $\lrcorner$