Theorem VII.15 Let X be a Banach space

Set $K = (B_{X^*}, w^*)$. Then K is a compact Hausdorff space

$J : X \to \mathcal{C}(K)$ defined by $J(x) = \varkappa(x)\restriction_K$,

i.e. $J(x)(x^*) = x^*(x)$, $x^* \in K$, is a linear isometry, $w \to \tau_p$ homeomorphism, $J(X)$ is τ_p-closed

Proof ① It is clear that J is a well-defined linear operator

② J is an isometry:

$$\|J(x)\| = \sup\{|x^*(x)|\, ;\, x^* \in B_{X^*}\} \overset{\text{dual formula for the norm}}{=} \|x\|$$

③ J is $w \to \tau_p$ cts

⌈Fix $x^* \in K = B_{X^*}$ then $J(x)(x^*) = x^*(x)$,
i.e. $(x \mapsto J(x)(x^*)) = x^*$, which is weakly cts ⌋

④ J^{-1} is $\tau_p \to w$ cts

⌈Fix $x^* \in X^*$. Find $c > 0$ s.t. $\frac{x^*}{c} \in B_{X^*}$.
Then for each $f \in J(X)$ we have
$$J^{-1}(f)(x^*) = c\, J^{-1}(f)\left(\tfrac{x^*}{c}\right) = c \cdot f\left(\tfrac{x^*}{c}\right),$$

so $f \mapsto J^{-1}(f)(x^*)$ is τ_p-cts ⌋

⑤ $\mathbb{F} = \mathbb{R} \Rightarrow J(X) = \{f \in \mathcal{C}(K);\ f \text{ affine}, \ \& \ f(0) = 0\}$

$\mathbb{F} = \mathbb{C} \Rightarrow J(X) = \{f \in \mathcal{C}(K);\ f \text{ affine},\ f(0) = 0$
$\& \ \forall \alpha \in \mathbb{C}, |\alpha| = 1 : f(\alpha x^*) = \alpha f(x^*)$
$\forall x^* \in K\}$

⌈Recall: f affine $\Leftrightarrow \forall\, x^*, y^* \in K \ \forall\, t \in [0,1]$
$$f(t x^* + (1-t) y^*) = t f(x^*) + (1-t) f(y^*)$$

"$\subset$" clear

"$\supset$" $f \in$ Right-handside $\Rightarrow \exists\, g: X^* \to \mathbb{F}$ linear

$g \restriction K = f$

$\shortparallel$

$g \restriction B_{X^*}$

By Corollary 15 $f \in \mathscr{X}(+)$,

so $g \in J(X)$

And, it is clear that the subspaces on the RHS are τ_p-closed $\lrcorner$