V/1.

<u>Proof of Theorem 34</u> X Banach space, $K \subset X$ weakly compact $\Rightarrow \overline{\mathrm{aco}}\, K$ is weakly compact

<u>Proof:</u> Let $T : X^* \to C(K)$ be defined by $T(x^*) = x^*\restriction K$

Then (similarly as in the proof of Theorem 33 (i) $\Rightarrow$ (ii)):

① T is a bdd linear operator, $\|T\| \leq \sup\{\|x\|, x \in K\}$.

② T is $w^* \to \tau_p$ cts, so $T(B_{X^*})$ is τ_p-compact. Since it is bdd, it is weakly compact, so T is weakly compact.

③ It follows from Theorem 33 (i) $\Rightarrow$ (iii) that T' is $w^* \to w$ cts

④ $x \in K \Rightarrow$ define $\delta_x \in C(K)^*$ by $\delta_x(f) = f(x)$.
[i.e., it is the Dirac measure supported at x]
Then $T'(\delta_x) = \varkappa(x)$
$[\, T'(\delta_x)(x^*) = \delta_x(Tx^*) = x^*(x) = \varkappa(x)(x^*) \,]$

⑤ $\overline{\mathrm{aco}\{\delta_x, x \in K\}}^{w^*} = B_{C(K)^*}$

[bipolar theorem: $\overline{\mathrm{aco}\{\delta_x, x \in K\}}^{w^*} = (\{\delta_x, x \in K\}_0)^0 =$
$= (B_{C(K)})^0 = B_{C(K)^*}$]

⑥ So, $T'(B_{C(K)^*}) = T'(\overline{\mathrm{aco}\{\delta_x, x \in K\}}^{w^*}) =$

$= \overline{T'(\mathrm{aco}\{\delta_x, x \in K\})}^{w} = \overline{\mathrm{aco}(\varkappa(K))}^{w} = \overline{\mathrm{aco}}(\varkappa(K))$

$\subset$ T' is w^*-w cts by ③

$\supset$ the LHS is weakly compact

It follows that $\overline{\mathrm{aco}}(\varkappa(K))$ is weakly compact, thus $\overline{\mathrm{aco}}(K)$ is weakly compact as well!