I. Determine and draw domains and contours of the following functions

1. $f(x,y) = x + \sqrt{y}$ **2.** $f(x,y) = \frac{y}{x}$ **3.** $f(x,y) = x^2 + y^2$ **4.** $f(x,y) = x^2 - y^2$
5. $f(x,y) = \sqrt{xy}$ **6.** $f(x,y) = \sqrt{1 - x^2 - y^2}$ **7.** $f(x,y) = \frac{1}{\sqrt{x^2+y^2-1}}$
8. $f(x,y) = \sqrt{(x^2 + y^2 - 1)(4 - x^2 - y^2)}$ **9.** $f(x,y) = \sqrt{1 - (x^2 + y)^2}$
10. $f(x,y) = \sqrt{\sin(x^2 + y^2)}$ **11.** $f(x,y) = \operatorname{sgn}(\sin x \cdot \sin y)$ **12.** $f(x,y) = |x| + y$

Decide whether the following sets are open or closed and determine their interior, closure and boundary

13. (a) $\mathbb{Q}$ (b) $\mathbb{N}$ (c) $\{\frac{1}{n} \mid n \in \mathbb{N}\}$ (d) $(-\infty, 0) \cup \{x \in \mathbb{Q} \mid x > 0\}$
14. $\{[x,y] \in \mathbb{R}^2 \mid x > 0,\ y \leq 0\}$ **15.** $\{[x,y] \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ **16.** $\{[x,y] \in \mathbb{R}^2 \mid x^2 + y^2 \geq 1\}$
17. $\{[x,y] \in \mathbb{R}^2 \mid x^2 + e^y > 17\}$ **18.** $\{[x,y] \in \mathbb{R}^2 \mid |x + y| > x + y\}$
19. $\{[x,y] \in \mathbb{R}^2 \mid |x - y| = x - y\}$ **20.** $\{[x,y] \in \mathbb{R}^2 \mid x^2 + y^2 + 2xy = 5\}$
21. $\{[x,y,z] \in \mathbb{R}^3 \mid x \geq 0, y > 0, x + y = 2, z \leq 0\}$

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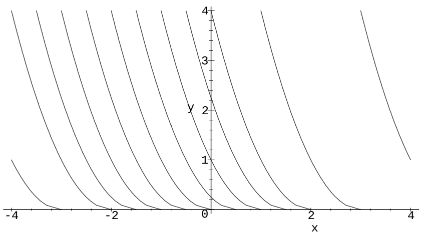
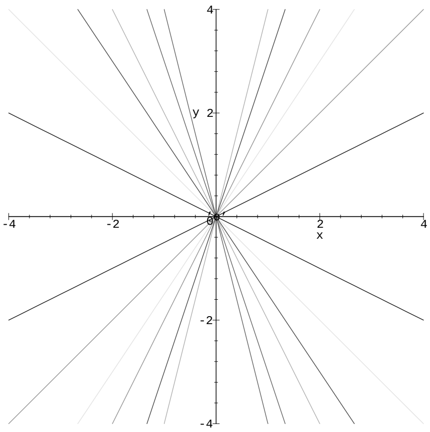
Answers and hints.

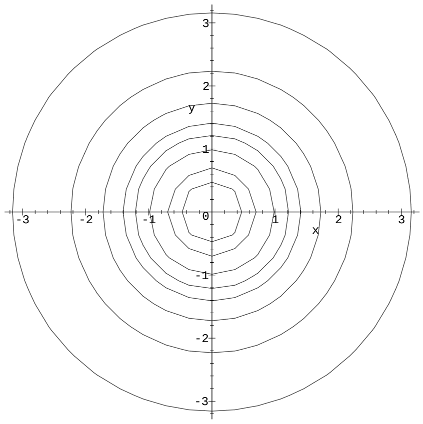
1. $D_f = \{[x,y] \in \mathbb{R}^2 \mid y \geq 0\}$



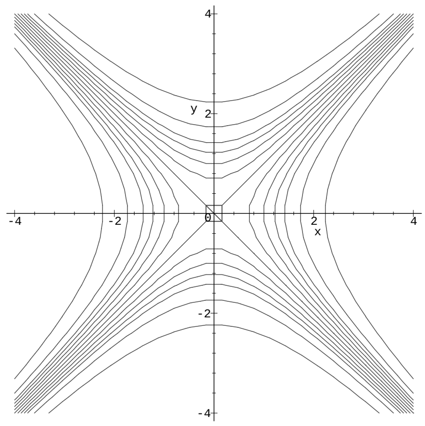
2. $D_f = \{[x,y] \in \mathbb{R}^2 \mid x \neq 0\}$



3. $D_f = \mathbb{R}^2$

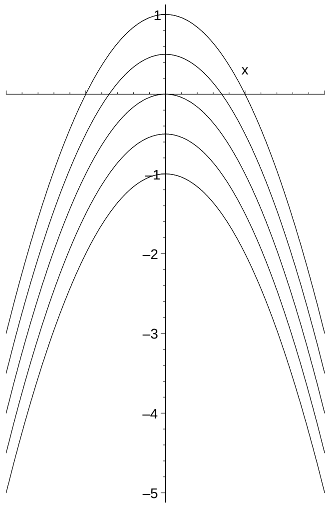


4. $D_f = \mathbb{R}^2$

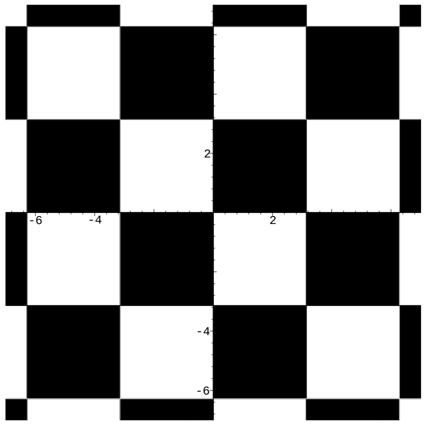


5. $D_f = \{[x,y] \mid (x \geq 0 \ \&\ y \geq 0) \vee (x \leq 0 \ \&\ y \leq 0)\}$, contours are hyperbolas of the form $y = \frac{c}{x}$ for $c > 0$, together with the pair of axes. **6.** $D_f = \{[x,y] \mid x^2 + y^2 \leq 1\}$, contours are circles. **7.** $D_f = \{[x,y] \mid x^2 + y^2 > 1\}$, contours are circles. **8.** $D_f = \{[x,y] \mid 1 \leq x^2 + y^2 \leq 4\}$, contours are pairs of circles, one of them is just a circle. **9.** $D_f = \{[x,y] \mid -x^2 - 1 \leq y \leq -x^2 + 1\}$, contours are pairs of parabolas, one of them is just a parabola. **10.** $D_f = \{[x,y] \mid 2k\pi \leq x^2 + y^2 \leq (2k+1)\pi$ for some $k = 0, 1, 2, \ldots\}$, contours are sequences of circles. **11.** $D_f = \mathbb{R}^2$, there are just three contours - one of them is formed by the interiors of black circles, the second one by interiors of white circles and the third one by boundary lines. **12.** $D_f = \mathbb{R}^2$, contours are graphs of functions $y = k - |x|$.

Ad **9.**



Ad **11.**



13. (a) $\operatorname{int}\mathbb{Q} = \emptyset$, $\operatorname{bd}\mathbb{Q} = \overline{\mathbb{Q}} = \mathbb{R}$, $\mathbb{Q}$ is neither open nor closed. (b) $\mathbb{N}$ is closed, $\operatorname{int}\mathbb{N} = \emptyset$, $\operatorname{bd}\mathbb{N} = \overline{\mathbb{N}} = \mathbb{N}$ (c) The set is neither open nor closed, the interior is empty, both boundary and closure are $\{\frac{1}{n} \mid n \in \mathbb{N}\} \cup \{0\}$. (d) The set is neither open nor closed, the interior is $(-\infty, 0)$, the closure is $\mathbb{R}$, the boundary is $[0, \infty)$. **14.** The set is neither open nor closed, the interior is $\{[x, y] \in \mathbb{R}^2 \mid x > 0,\ y < 0\}$, the closure is $\{[x, y] \in \mathbb{R}^2 \mid x \leq 0,\ y \leq 0\}$, the boundary is $\{[x, y] \in \mathbb{R}^2 \mid x \leq 0 \;\&\; y \leq 0 \;\&\; (x = 0 \vee y = 0)\}$. **15.** Open; closure $\{[x, y] \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$, boundary $\{[x, y] \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. **16.** Closed; interior $\{[x, y] \in \mathbb{R}^2 \mid x^2 + y^2 > 1\}$, boundary $\{[x, y] \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. **17.** Open; boundary $\{[x, y] \in \mathbb{R}^2 \mid x^2 + e^y = 17\}$, closure $\{[x, y] \in \mathbb{R}^2 \mid x^2 + e^y \leq 17\}$ **18.** Open; closure $\{[x, y] \mid x + y \leq 0\}$, boundary $\{[x, y] \mid x + y = 0\}$. **19.** Closed; interior $\{[x, y] \mid x + y > 0\}$, boundary $\{[x, y] \mid x + y = 0\}$. **20.** Closed with empty interior. **21.** Neither open nor closed. Empty interior. Boundary and closure $\{[x, y, z] \in \mathbb{R}^3 \mid x \geq 0, y \geq 0, x + y = 2, z \leq 0\}$.