II. Compute partial derivatives of the following functions at all points where they exist

1. $x^m y^n$ **2.** e^{xy} **3.** $xy + yz + zx$ **4.** $\sqrt{x^2+y^2}$ **5.** $\sqrt[3]{x^3+y^3}$ **6.** $|x|\cdot|y|$ **7.** $\sqrt[3]{xy}$ **8.** $|y-\sin x|$ **9.** $|\sin y - \sin x|$ **10.** $\sqrt[3]{x+y^2}$ **11.** $f(x,y) = e^{\frac{-1}{x^2+xy+y^2}}$, $f(0,0)=0$ **12.** $\left(\frac{x}{y}\right)^z$ **13.** $x^{\frac{y}{z}}$ **14.** x^{y^z}

Answers and hints. **1.** $\frac{\partial f}{\partial x} = mx^{m-1}y^n$, $\frac{\partial f}{\partial y} = nx^m y^{n-1}$ for $(x,y)\in\mathbb{R}^2$. **2.** $\frac{\partial f}{\partial x} = ye^{xy}$, $\frac{\partial f}{\partial y} = xe^{xy}$ for $(x,y)\in\mathbb{R}^2$. **3.** $\frac{\partial f}{\partial x} = y+z$, $\frac{\partial f}{\partial y} = x+y$, $\frac{\partial f}{\partial z} = x+y$ for $(x,y,z)\in\mathbb{R}^3$. **4.** $\frac{\partial f}{\partial x}(x,y) = \frac{x}{\sqrt{x^2+y^2}}$, $\frac{\partial f}{\partial y}(x,y) = \frac{y}{\sqrt{x^2+y^2}}$, for $(x,y)\neq(0,0)$. $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ do not exist. **5.** $\frac{\partial f}{\partial x}(x,y) = \frac{x^2}{\sqrt[3]{x^3+y^3}}$, $\frac{\partial f}{\partial y}(x,y) = \frac{y^2}{\sqrt[3]{x^3+y^3}}$ for $y\neq -x$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 1$, $\frac{\partial f}{\partial x}(x,-x)$ and $\frac{\partial f}{\partial y}(x,-x)$ do not exist for $x\neq 0$. **6.** $\frac{\partial f}{\partial x}(x,y) = |y|\operatorname{sgn} x$ for $x\neq 0$. $\frac{\partial f}{\partial y}(x,y) = |x|\operatorname{sgn} y$ for $y\neq 0$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$. $\frac{\partial f}{\partial x}(0,y)$ for $y\neq 0$ and $\frac{\partial f}{\partial y}(x,0)$ for $x\neq 0$ do not exist. **7.** $\frac{\partial f}{\partial x}(x,y) = \frac{\sqrt[3]{y}}{3\sqrt[3]{x^2}}$ for $x\neq 0$. $\frac{\partial f}{\partial y}(x,y) = \frac{\sqrt[3]{x}}{3\sqrt[3]{y^2}}$ for $y\neq 0$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$. $\frac{\partial f}{\partial x}(0,y)$ pro $y\neq 0$ and $\frac{\partial f}{\partial y}(x,0)$ for $x\neq 0$ do not exist. **8.** $\frac{\partial f}{\partial x}(x,y) = -\operatorname{sgn}(y-\sin x)\cdot\cos x$, $\frac{\partial f}{\partial y}(x,y) = \operatorname{sgn}(y-\sin x)$, for $y\neq \sin x$. $\frac{\partial f}{\partial y}(x,\sin x)$ does not exist for $x\in\mathbb{R}$. $\frac{\partial f}{\partial x}(\frac{\pi}{2}+k\pi, (-1)^k) = 0$ for $k\in\mathbb{Z}$. $\frac{\partial f}{\partial x}(x,\sin x)$ does not exist for $x\neq\frac{\pi}{2}+k\pi$. **9.** $\frac{\partial f}{\partial x}(x,y) = \cos x\operatorname{sgn}(\sin x - \sin y)$, $\frac{\partial f}{\partial y}(x,y) = -\cos y\operatorname{sgn}(\sin x-\sin y)$, for $\sin x\neq\sin y$. $\frac{\partial f}{\partial x}(\frac{\pi}{2}+k\pi, \frac{\pi}{2}+l\pi) = \frac{\partial f}{\partial y}(\frac{\pi}{2}+k\pi, \frac{\pi}{2}+l\pi) = 0$. At remaining points partial derivatives do not exist. **10.** $\frac{\partial f}{\partial x}(x,y) = \frac{1}{3\sqrt[3]{x+y^2}}$, $\frac{\partial f}{\partial y}(x,y) = \frac{2y}{3\sqrt[3]{x+y^2}}$, for $x\neq -y^2$, $\frac{\partial f}{\partial x}(-x^2,x)$ and $\frac{\partial f}{\partial y}(-x^2,x)$ do not exist for $x\in\mathbb{R}$. **11.** $\frac{\partial f}{\partial x} = e^{\frac{-1}{x^2+xy+y^2}}\cdot\frac{2x+y}{(x^2+xy+y^2)^2}$, $\frac{\partial f}{\partial y} = e^{\frac{-1}{x^2+xy+y^2}}\cdot\frac{x+2y}{(x^2+xy+y^2)^2}$ for $(x,y)\neq(0,0)$; at $(0,0)$ both partial derivatives are zero. **12.** If $x,y>0$ or $x,y<0$, then $\frac{\partial f}{\partial x} = \frac{z}{y}\cdot\left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial y} = -\frac{zx}{y^2}\cdot\left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^z\cdot\log\frac{x}{y}$. **13.** If $x>0$ and $y\neq 0$, then $\frac{\partial f}{\partial x} = \frac{y}{z}\cdot x^{\frac{y}{z}-1}$; $\frac{\partial f}{\partial y} = x^{\frac{y}{z}}\cdot\log x\cdot\frac{1}{z}$; $\frac{\partial f}{\partial z} = -x^{\frac{y}{z}}\cdot\log x\cdot\frac{y}{z^2}$. **14.** If $x,y>0$, then $\frac{\partial f}{\partial x} = y^z\cdot x^{y^z-1}$; $\frac{\partial f}{\partial y} = x^{y^z}\cdot\log x\cdot zy^{z-1}$; $\frac{\partial f}{\partial z} = x^{y^z}\cdot\log x\cdot y^z\cdot\log y$.