

## Definition of submatrices $\mathbb{A}_{ij}$

$$\mathbb{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

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# Determinant of a 2-by-2 matrix

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} =$$

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$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (-1)^{1+1} \cdot a_{11}$$

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$$\left| \begin{array}{cc} & a_{22} \\ a_{22} & \end{array} \right| = (-1)^{1+1} \cdot a_{11} \cdot a_{22}$$

# Determinant of a 2-by-2 matrix

$$\begin{vmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & a_{22} \end{vmatrix} = (-1)^{1+1} \cdot a_{11} \cdot a_{22}$$

# Determinant of a 2-by-2 matrix

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (-1)^{1+1} \cdot a_{11} \cdot a_{22} + (-1)^{2+1}$$

# Determinant of a 2-by-2 matrix

$$\begin{vmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & a_{22} \end{vmatrix} = (-1)^{1+1} \cdot a_{11} \cdot a_{22} + (-1)^{2+1} \cdot \color{red}{a_{21}}$$

# Determinant of a 2-by-2 matrix

$$\left| \begin{array}{c} a_{12} \\ \end{array} \right| = (-1)^{1+1} \cdot a_{11} \cdot a_{22} + (-1)^{2+1} \cdot a_{21} \cdot \textcolor{red}{a_{12}}$$

## Determinant of a 2-by-2 matrix

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (-1)^{1+1} \cdot a_{11} \cdot a_{22} + (-1)^{2+1} \cdot a_{21} \cdot a_{12} = a_{11}a_{22} - a_{21}a_{12}$$

# Determinant of a 3-by-3 matrix

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+1} a_{21} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \color{red}{a_{31}} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+1} a_{21} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{3+1} a_{31} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

# Determinant of a 3-by-3 matrix

$$\begin{aligned} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} &= (-1)^{1+1} a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \\ &\quad + (-1)^{2+1} a_{21} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{3+1} a_{31} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ &= \color{red}{a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{21}a_{12}a_{33}} \\ &\quad \color{red}{+ a_{21}a_{13}a_{32} + a_{31}a_{12}a_{23} - a_{31}a_{13}a_{22}} \end{aligned}$$

## Determinant of a 3-by-3 matrix – Sarrus' scheme

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$$

## Determinant of a 3-by-3 matrix – Sarrus' scheme

$$\begin{vmatrix} \color{red}{a_{11}} & a_{12} & a_{13} \\ a_{21} & \color{red}{a_{22}} & a_{23} \\ a_{31} & a_{32} & \color{red}{a_{33}} \end{vmatrix}$$
$$\begin{array}{l} a_{11} \quad a_{12} \quad a_{13} \\ a_{21} \quad a_{22} \quad a_{23} \\ a_{31} \quad a_{32} \quad a_{33} \end{array}$$
$$= \color{red}{a_{11}a_{22}a_{33}}$$

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$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$$

$$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \color{red}{a_{31}} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{matrix} a_{11} & \color{red}{a_{12}} & a_{13} \\ a_{21} & a_{22} & \color{red}{a_{23}} \end{matrix}$$

$$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + \color{red}{a_{31}a_{12}a_{23}}$$

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$$\begin{vmatrix} a_{11} & a_{12} & \color{red}{a_{13}} \\ a_{21} & \color{red}{a_{22}} & a_{23} \\ \color{red}{a_{31}} & a_{32} & a_{33} \end{vmatrix}$$
$$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - \color{red}{a_{13}a_{22}a_{31}}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & \color{red}{a_{23}} \\ a_{31} & \color{red}{a_{32}} & a_{33} \\ \hline \color{red}{a_{11}} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}$$

$$\begin{aligned} &= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} \\ &\quad - a_{13}a_{22}a_{31} - \color{red}{a_{23}a_{32}a_{11}} \end{aligned}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & \color{red}{a_{33}} \\ \hline a_{11} & \color{red}{a_{12}} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}$$

$$\begin{aligned} &= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} \\ &\quad - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - \color{red}{a_{33}a_{12}a_{21}} \end{aligned}$$