① $y(n+2)-y(n+1)-y(n)=0$

$y(1)=y(2)=1$

$\chi(\lambda)=\lambda^2-\lambda-1$

Kořeny: $\dfrac{1\pm\sqrt{1+4}}{2}=\dfrac{1\pm\sqrt{5}}{2}$

F.S.: $\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n\right\}_{n=1}^{\infty},\ \left\{\left(\frac{1-\sqrt{5}}{2}\right)^n\right\}_{n=1}^{\infty}$

řešení: $y(n)=a\cdot\left(\frac{1+\sqrt{5}}{2}\right)^n+b\cdot\left(\frac{1-\sqrt{5}}{2}\right)^n$

$a,b\in\mathbb{R}$

$y(1)=1,\ y(2)=1$ ⇒ [illegible] dostaneme C

$a=\frac{1}{\sqrt{5}},\quad b=-\frac{1}{\sqrt{5}}$

$$y(n)=\frac{1}{\sqrt{5}}\underbrace{\Big(\underbrace{\frac{1+\sqrt{5}}{2}}_{>1}\Big)^n}_{\to+\infty}-\frac{1}{\sqrt{5}}\overbrace{\Big(\underbrace{\frac{1-\sqrt{5}}{2}}_{\in(-1,0)}\Big)^n}^{\to 0}$$

② $y(n+3)-y(n+2)-y(n+1)+y(n)=0$

$y(1)=1,\ y(2)=1,\ y(3)=2$

$\chi(\lambda)=\lambda^3-\lambda^2-\lambda+1=\lambda^2(\lambda-1)-(\lambda-1)$

$=(\lambda^2-1)(\lambda-1)=(\lambda-1)^2(\lambda+1)$

F.S. $\{1^n\}$, $\{n \cdot 1^n\}$, $\{(-1)^n\}$

obecné řešení: $y(n) = a + bn + c \cdot (-1)^n$

$$\begin{aligned} a + b - c &= 1 \\ a + 2b + c &= 1 \\ a + 3b - c &= 2 \end{aligned} \qquad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 3 & -1 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 1 \end{array}\right) \qquad b = \frac{1}{2} \quad c = -\frac{1}{2}b = -\frac{1}{4}$$

$$a = 1 - b + c = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$y(n) = \frac{1}{4} + \frac{1}{2}n - \frac{1}{4} \cdot (-1)^n \quad \to +\infty$$

(3) $y(n+3) - y(n+2) - y(n+1) + y(n) = -1$

kořeny ch. p.: 1 nás. 2, -1 nás. 1

$y(n) = a + bn + c \cdot (-1)^n$

$$-1 = 1^n \cdot (-1 \cdot \cos n \cdot 0 + 1 \cdot \sin n \cdot 0)$$

$\alpha = 1, \beta = 0$ .... $\xi = 1$ kořen nás. 2

$y_p(n) = n^2 d \cdot 1$ $\quad (n^2 \cdot 1^k \cdot \underline{d})$ $\qquad P(n) = -1$ stupeň 0

polynom R

$$d \cdot (n+3)^2 - d \cdot (n+2)^2 - d \cdot (n+1)^2 + d \cdot n^2 = -1$$

$$d \cdot (n^2 + 6n + 9 - n^2 - 4n - 4 - n^2 - 2n - 1 + n^2) = 4d$$

$y(n) = \boxed{-\frac{1}{4}n^2 + a + bn + c \cdot (-1)^n} \to -\infty$ $\qquad d = -\frac{1}{4}$