Written Exam on Mathematics II for IES FSV UK (C)

Summer Semester 2012-2013

Problem 1: Compute the inverses of the matrices $\mathbb{A}$ and $\mathbb{B}$, where $\mathbb{A}$ is given below and $\mathbb{B}$ has in the first row 3-tuple of the third row of $\mathbb{A}$, in the second row 10-tuple of the first row of $\mathbb{A}$, in the third row the second row of $\mathbb{A}$ and in the fourth row the fourth row of $\mathbb{A}$.

$$\mathbb{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{pmatrix}$$

(10 points)

Problem 2: Determine and draw the domain of the function

$$f(x,y) = \sqrt{y^2 - 4x^2},$$

compute its partial derivatives with respect to all the variables at all points where they exist.

(10 points)

Problem 3: Let us consider the equation $\arcsin(x+y) + \sin(x+y^2) = 0$ and the point $[-1,1]$. Show that this equation defines a C^∞ function $y = f(x)$ defined on a neighborhood of -1, which satisfies $f(-1) = 1$. Compute $f'(-1)$, $f''(-1)$ and determine the equation of the tangent line to the graph of f at the point $[-1, f(-1)]$.

(10 points)

Problem 4: Determine sup and inf of the function f on the set M and decide whether these values are attained, if

$$f(x,y,z) = x - y \text{ and } M = \left\{ [x,y,z] \in \mathbb{R}^3 : x^2 + 2y^2 + z^2 = 4, xy + \sqrt{\frac{3}{2}} \geq 0 \right\}$$

(15 points)

Problem 5: Compute the following antiderivative on maximal possible intervals:

$$\int \frac{5x^4}{(x^2 - 2x + 2)(x^2 - x - 2)} \, \mathrm{d}x$$

(15 points)

Answers to the Written Exam on Mathematics II for IES FSV UK (C)
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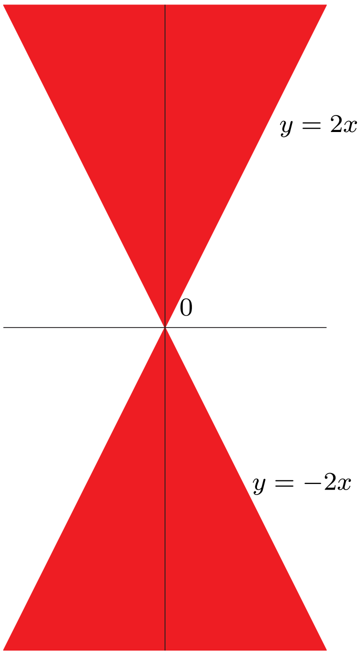
Problem 1: $\mathbb{A}^{-1} = \begin{pmatrix} 4 & -6 & 4 & -1 \\ -\frac{13}{3} & \frac{19}{2} & -7 & \frac{11}{6} \\ \frac{3}{2} & -4 & \frac{7}{2} & -1 \\ -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \end{pmatrix}$, $\mathbb{B}^{-1} = \begin{pmatrix} \frac{4}{3} & \frac{2}{5} & -6 & -1 \\ -\frac{7}{3} & -\frac{13}{30} & \frac{19}{2} & \frac{11}{6} \\ \frac{7}{6} & \frac{3}{20} & -4 & -1 \\ -\frac{1}{6} & -\frac{1}{60} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$.

Problem 2: $D_f = \{[x,y] \in \mathbf{R}^2 : (y \geq 0 \ \& \ -\frac{y}{2} \leq x \leq \frac{y}{2}) \text{ or } (y \leq 0 \ \& \ \frac{y}{2} \leq x \leq -\frac{y}{2})\}$.

Picture of the domain:



$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2\sqrt{y^2-4x^2}} \cdot (-8x)$ and $\frac{\partial f}{\partial y}(x,y) = \frac{1}{2\sqrt{y^2-4x^2}} \cdot 2y$; both partial derivatives on the set $\{[x,y] \in \mathbf{R}^2 : (y > 0 \ \& \ -\frac{y}{2} < x < \frac{y}{2}) \text{ or } (y < 0 \ \& \ \frac{y}{2} < x < -\frac{y}{2})\}$. At points $[x, 2x]$ and $[x, -2x]$, $x \in \mathbf{R} \setminus \{0\}$ the partial derivatives have no sense, since there is neither horizontal nor vertical segment centered at that point and contained in D_f. At $[0,0]$ the partial derivative with respect to x has no sense, as no horizontal segment centered at $[0,0]$ belongs to D_f. $\frac{\partial f}{\partial y}(0,0)$ does not exist.

Problem 3: $f'(-1) = -\frac{2}{3}$, $f''(-1) = -\frac{8}{27}$, tangent line $y = 1 - \frac{2}{3}(x+1)$.

Problem 4: Maximum $\sqrt{3} + \frac{1}{\sqrt{2}}$ at the point $[\sqrt{3}, -\frac{1}{\sqrt{2}}, 0]$; minimum $-\sqrt{3} - \frac{1}{\sqrt{2}}$ at the point $[-\sqrt{3}, \frac{1}{\sqrt{2}}, 0]$.

Problem 5: $\int \frac{5x^4}{(x^2-2x+2)(x^2-x-2)}\,\mathrm{d}x \overset{c}{=} 5x - \frac{1}{3}\log|x+1| + \frac{40}{3}\log|x-2| + \log(x^2-2x+2) + 6\,\mathrm{arctg}(x-1)$ on each of the three intervals $(-\infty,-1)$, $(-1,2)$ and $(2,+\infty)$.