MA341

Petr Kaplický

February 25, 2020

Plan

Week 5, lecture 1

$$y(t) = y_p(t) + c_1 y_1(t) + c_2 y_2(t) \tag{5}$$

Existence and Uniqueness: Nonhomogeneous Case

Theorem 4. For any real numbers $a(\neq 0)$, b, c, t_0, Y_0, and Y_1, suppose $y_p(t)$ is a particular solution to (3) in an interval I containing t_0 and that $y_1(t)$ and $y_2(t)$ are linearly independent solutions to the associated homogeneous equation (4) in I. Then there exists a unique solution in I to the initial value problem

$$ay'' + by' + cy = f(t)\ , \qquad y(t_0) = Y_0,\ y'(t_0) = Y_1\ , \tag{6}$$

and it is given by (5), for the appropriate choice of the constants c_1, c_2.

Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = Ct^m e^{rt} ,$$

where m is a nonnegative integer, use the form

$$y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{rt} , \tag{14}$$

with

(i) $s = 0$ if r is not a root of the associated auxiliary equation;
(ii) $s = 1$ if r is a simple root of the associated auxiliary equation; and
(iii) $s = 2$ if r is a double root of the associated auxiliary equation.

To find a particular solution to the differential equation

$$ay'' + by' + cy = \begin{cases} Ct^m e^{\alpha t}\cos\beta t \\ \quad\text{or} \\ Ct^m e^{\alpha t}\sin\beta t \end{cases}$$

for $\beta \neq 0$, use the form

$$\begin{aligned} y_p(t) = {} & t^s(A_m t^m + \cdots + A_1 t + A_0)e^{\alpha t}\cos\beta t \\ & + t^s(B_m t^m + \cdots + B_1 t + B_0)e^{\alpha t}\sin\beta t , \end{aligned} \tag{15}$$

with

(iv) $s = 0$ if $\alpha + i\beta$ is not a root of the associated auxiliary equation; and
(v) $s = 1$ if $\alpha + i\beta$ is a root of the associated auxiliary equation.

Method of Variation of Parameters

To determine a particular solution to $ay'' + by' + cy = f$:

(a) Find two linearly independent solutions $\{y_1(t), y_2(t)\}$ to the corresponding homogeneous equation and take

$$y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t) \ .$$

(b) Determine $v_1(t)$ and $v_2(t)$ by solving the system in (9) for $v_1'(t)$ and $v_2'(t)$ and integrating.

(c) Substitute $v_1(t)$ and $v_2(t)$ into the expression for $y_p(t)$ to obtain a particular solution.