Lemma: Ať $A=(a^{ij})$ je symetrická a splňuje pro j. $\vartheta>0$:

$$\sum_{ij} a^{ij}\xi_i\xi_j \geq \vartheta|\xi|^2 \quad \text{pro lib. } \xi$$

Pak platí:

$$? \quad \sum_{ijkl} a^{ij}a^{kl}\, v_{ik}v_{jl} \geq \vartheta^2|v|^2, \quad \text{kde}$$

$$v_{ik} = \frac{\partial^2 u}{\partial x_i \partial x_k} \quad ?$$

Pokus o důkaz:

$V=(v_{ik})$ je diag. $\quad \exists P$ ON: $D=PVP^T$ a D diag

$D=\operatorname{diag}(d_k)$; $\quad V=P^TDP$

$$v_{ik}=\sum_\alpha P^T_{i\alpha}\, d_\alpha\, P_{\alpha k} = \sum_\alpha P_{\alpha i}\, d_\alpha\, P_{\alpha k}$$

$$v_{jl}=\sum_\beta P_{\beta j}\, d_\beta\, P_{\beta l}$$

$$\sum_{ijkl\alpha\beta} \underline{a}^{ij}\,\underline{a}^{kl}\, P_{\underline{\beta j}}\, d_\beta\, P_{\beta l}\, \underline{P}_{\underline{\alpha i}}\, d_\alpha\, P_{\underline{\alpha k}} =$$

$$\sum_{\alpha\beta}\Big(\sum_{ij} P_{\alpha i}\, a^{ij}\, P^T_{j\beta}\Big)\Big(\sum_{kl} P_{\alpha k}\, a^{kl}\, P^T_{l\beta}\Big)\, d_\alpha d_\beta = \sum_{\alpha\beta} M^2_{\alpha\beta}\, d_\alpha d_\beta$$

$$M=\Big(\sum_{ij} P_{\alpha i}\, a^{ij}\, P^T_{j\beta}\Big)_{\alpha\beta} \qquad \overset{?}{\geq} \vartheta^2|d|^2$$

$$M = PAP^T, \quad M^2 = PA^2P^T$$

Najděte správný důkaz!