Dk: Thm 1.2:

- $u \in C^{0,\alpha}(\Omega) \Rightarrow u \in \mathcal{L}^{\infty}(\Omega)$

$$\rho^{-\lambda}\int_{\Omega(x_0,\rho)} |u-(u)|^p = \rho^{-\lambda}\int_{\Omega(x_0,\rho)} \left|\int_{\Omega(x_0,\rho)} u(x)-u(y)\,dy\,\frac{1}{|\Omega(x_0,\rho)|}\right|^p dx$$

$$(u) = \frac{1}{|\Omega_{x_0,\rho}|}\int_{\Omega(x_0,\rho)} u(y)\,dy$$

Hölder/Jensen: $\left(\fint_{\Omega} f\right)^p \le \fint_{\Omega} f^p$

$$\le \rho^{-\lambda}\int_{\Omega(x_0,\rho)}\fint_{\Omega(x_0,\rho)} |u(x)-u(y)|^p\,dy\,dx$$

$$[u]_{0,\alpha}^p\,|x-y|^{\alpha p} \le c\,[u]^p_{0,\alpha}\,\rho^{\alpha p}$$

$$\le c\cdot \underbrace{\rho^{-\lambda+\alpha p+n}}_{=1}\,[u]^p_{0,\alpha} \le c\,[u]^p_{0,\alpha}$$

$$\Rightarrow [u]_{\mathcal{L}^{p,\lambda}} \le c\,[u]_{0,\alpha}$$

- $\boxed{u\in\mathcal{L}^{p,\lambda}(\Omega)}$ Pro pevné $x\in\Omega$, $R>0$

$$\{u_{x,R\cdot 2^{-n}}\}_{n=1}^{+\infty} \qquad u_{x,R\cdot 2^{-n}} =: u_n(x)$$

$\{u_n\}$ je ~~(loc.)~~ stejn. cauchy v Ω

$$|u_N-u_m| \le c_2\,[u]_{p,\lambda}\,(R\cdot 2^{-m})^{\frac{\lambda-n}{p}}; \quad m<N.$$

$$\Rightarrow u_n(x) \xrightarrow[n\to+\infty]{} \tilde{u}(x) = u(x) \text{ s.v. } x$$

Lebesgue o hustotě

$\tilde{u}$ je representant u v L^p .. tohle dále bereme.

- $f: x \to u_n(x)$ je spojitá na Ω

$$\left.\begin{array}{l} u_n \rightrightarrows \tilde{u} \\ \uparrow_{\text{spojité}} \end{array}\right\} \Rightarrow \tilde{u} \text{ je spojitá na } \Omega$$

Hlavní odhad normy:

- $\tilde{u}$ nezávisí na volbě R.

- $|u_{x,R} - \tilde{u}(x)| \le c_2 [u]_{p,\lambda} R^{\frac{\lambda - n}{p}}$

- Fix $x, y \in \Omega$: $|\tilde{u}(x) - \tilde{u}(y)| \le$

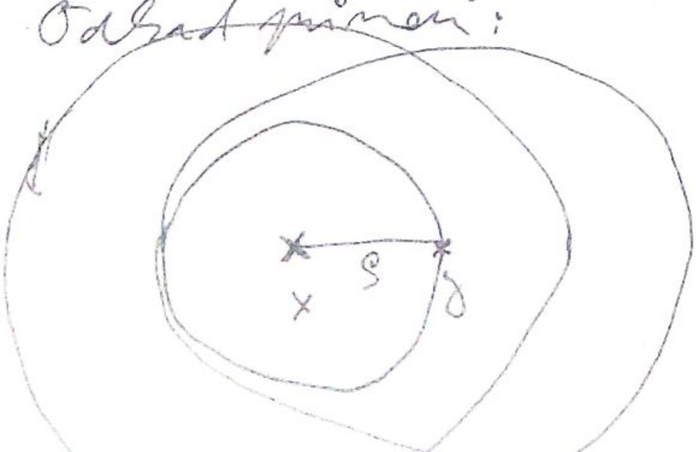
$|\tilde{u}(x) - u_{x,\varrho}| + |u_{x,\varrho} - u_{y,\varrho}| + |\tilde{u}(y) - u_{y,\varrho}|$

$\le c_4 [u]_{p,\lambda} \varrho^{\frac{\lambda - n}{p}}$. Volba $\varrho := 2|x-y|$

~~Odhad prostředí~~:

$\Omega(x,\varrho) \subset \Omega(x,2\varrho) \cap \Omega(y,2\varrho)$

$B(x,\varrho) \subset B(x,2\varrho) \cap B(y,2\varrho)$



$|u_{x,2\varrho} - u_{y,2\varrho}|^p \le C \left(|u_{x,2\varrho} - u(z)|^p + |u(z) - u_{y,2\varrho}|^p \right)$

$z \in \Omega$

$\fint_{B(x,\frac{\varrho}{2})} dz$

$|\tilde{u}_{x,2\varrho} - \tilde{u}_{y,2\varrho}|^p \le \frac{1}{|B(x,\frac{\varrho}{2})|} \left[\int_{\Omega \cap B(x,2\varrho)} |u_{x,2\varrho} - u(z)|^p dz + \int_{\Omega \cap B(y,2\varrho)} |u(z) - u_{y,2\varrho}|^p \right]$

$$|\tilde{u}_{x,\rho} - \tilde{u}_{y,\rho}|^p \le \frac{c}{\rho^n}\,\rho^{\lambda}\,[u]^p_{\mathcal{L}^{p,\lambda}}$$

$$|\tilde{u}_{x,\rho} - \tilde{u}_{y,\rho}| \le c\,\rho^{\frac{\lambda-n}{p}}\,[u]^p_{\mathcal{L}^{p,\lambda}}$$

$$\Rightarrow\ |u(x)-u(y)| \le c\,[u]_{\mathcal{L}^{p,\lambda}(\Omega)}\,|x-y|^{\frac{\lambda-n}{p}}$$

$\perp$.

Druhý přechod opraven.

- $u \in L^\infty(\Omega)$

$$|u(x)| \le \underbrace{\Big|u(x) - u_{x,\overset{1}{R}}\Big|}_{\text{odhad} \le c[u]_{p,\lambda}} + \Big|u_{x,\overset{1}{R}}\Big|$$

$$\Big|\fint_{\Omega(x,1)} u\Big| \le \Big(\fint_{\Omega(x,1)} |u|^p\Big)^{1/p} \le \|u\|_{\mathcal{L}^{p,\lambda}}$$

$\perp$

Důsledky: Věta o vnoření:

Dk:

$$\int_{B(x,R)} |u-(u)| \underset{\text{Poincaré}}{\le} CR\int_{B(x,R)} |\nabla u| \le CR\left(\int_{B(x,R)} |\nabla u|^p\right)^{1/p} (R^n)^{\frac{1}{p'}}$$

$$\le C\cdot R^{1+n-\frac{n}{p}} \|u\|_{W^{1,p}}$$

$$\Rightarrow u \in \mathcal{L}^{1,\underbrace{n+1-\frac{n}{p}}_{\lambda}}(\Omega) \overset{\text{Věta}}{\Rightarrow} u \in C^{0,\alpha}(\bar{\Omega})$$

$$\alpha = \frac{n+1-\frac{n}{p}-n}{1} = 1-\frac{n}{p}$$

$\perp$.

Důsledek (Morrey): $p<n$

Jako nahoře:

$$\int_{B(x,R)} |u-(u)| \le CR^{n+\cancel{1}-\cancel{\frac{n}{p}}+\frac{\cancel{n}-\cancel{p}+p\varepsilon}{p}} \|\nabla u\|_{L^{p,\lambda}}$$

$\lambda = n-p+p\varepsilon$.

$$\le CR^{n+\varepsilon}\|\nabla u\|_{L^{p,\lambda}}.$$

$\perp$

Dk (Gårding)

1) $A_{ij}^{\alpha\beta}$ je konstantní

Fourier

~~[illegible]~~

$$\int_\Omega A_{ij}^{\alpha\beta}\,\partial_\beta u^j\,\partial_\alpha u^i \overset{\text{Parseval}}{=} A_{ij}^{\alpha\beta}\int_\Omega \widehat{\partial_\beta u^j}\;\widehat{\partial_\alpha u^i} = \int_\Omega A_{ij}^{\alpha\beta}\,\xi_\beta \hat{u}^j\,\xi_\alpha \hat{u}^i\,(2\pi i)^2$$

$$\overset{\text{L-H}}{\geq} \int_\Omega \lambda\,|\xi|^2\,|\hat{u}|^2\,(2\pi i)^2 = \lambda(2\pi i)^2\int_\Omega \xi_k \xi_k \hat{u}^j \hat{u}^j$$

$$\overset{\text{Parseval}}{=} \lambda\int_\Omega \widehat{\partial_k u_j}\;\widehat{\partial_k u_j} = \lambda\int_\Omega |\nabla u|^2$$

2) ať $u \in W_0^{1,2}(\Omega)$ má nosič v $B(x_0,R)$

$$\int_\Omega A_{ij}^{\alpha\beta}(x)\,\partial_\beta u^j\,\partial_\alpha u^i = \int_\Omega A_{ij}^{\alpha\beta}(x_0)\,\partial_\beta u^j\,\partial_\alpha u^i + \left(A_{ij}^{\alpha\beta}(x) - A_{ij}^{\alpha\beta}(x_0)\right)(\dots)$$

$$\geq \lambda\int_\Omega |\nabla u|^2 - \omega(x_0,R)\cdot C\,|\nabla u|^2 \underset{\substack{\text{je-li}\\ \omega(x_0,R)\\ \text{malé}}}{\geq} \frac{\lambda}{2}\int_\Omega |\nabla u|^2$$

$$\omega(x_0,R) = \sup_{x,y\in B(x_0,R)} |A(x) - A(y)|$$

3) Rozklad 1 tak, aby na každé z množin bylo

$\omega(x_0, R)$ malé ← A je st. spojité

Přesněji: $\{U_j\}$; rozklad 1 ξ_j^2

$$\int_\Omega A_{ij}^{\alpha\beta}\, \partial_\beta u^j\, \partial_\alpha u^i\, \xi_k \xi_k \longrightarrow \text{opravit 2)}$$

- odhadnout člen nižšího řádu