26.10. – místnost 583

Prop. 2.1: Dk:

Vol η – cut-off funkce $\quad \eta \equiv 1$ na $B_R(x_0)$

$\eta \equiv 0$ vně $B_{2R}(x_0)$

$|\nabla \eta| \leq \frac{c}{R}$

Testuj: $\xi^2 u$

$$D_\alpha(\xi^2 u^i) = D_\alpha \xi^2 u^i + \xi^2 D_\alpha u^i$$

$$\int_\Omega A^{\alpha\beta}_{ij} D_\beta u^j D_\alpha u^i \xi^2 \leq \left|\int_\Omega A^{\alpha\beta}_{ij} D_\beta u^j 2 D_\alpha \xi \cdot \xi u^i\right|$$

$$\leq c \int_\Omega |\nabla u| \cdot \xi \, |u| \cdot \frac{1}{R}$$

(LS): $\displaystyle\int_\Omega A^{\alpha\beta}_{ij} D_\beta(u^j \xi) D_\alpha(u^i \xi) + \text{zbytek}$

$$\geq c \int_\Omega |\nabla(u\xi)|^2 - |\text{zbytek}|$$

$$\geq c \int_\Omega |\nabla u|^2 \cdot \xi^2 - |\text{zbytek}|$$

Dohromady: $\displaystyle c \int_\Omega |\nabla u|^2 \xi^2 \leq C \int_\Omega |u|^2 \frac{1}{R^2} \chi_{\operatorname{supp}(\nabla \xi)}$

L-H podm.:

$$A_{ij}^{\alpha\beta}\xi_\alpha\xi_\beta\eta_i\eta_j \geq \nu|\xi|^2|\eta|^2 \quad \forall\eta,\xi$$

DR Theorem 2.1:

Pokud $\rho\sim R$. Odhady triv.

K(2.7): $\displaystyle\int_{B_\rho}|u|^2 \leq C\|u\|^2_{L^\infty(B_\rho)}\rho^n \leq C\|u\|^2_{W^{k,2}(B_\rho)}\rho^n$ ($\rho<\frac{R}{2}$)

$\leq C\rho^n\displaystyle\int_{B_R}|u|^2 \xrightarrow{\text{Škálování}} C(R)=\frac{C}{R^n}$

Caccioppoli

$C=C(R)$

(2.7)

K(2.8): $\displaystyle\int_{B_\rho}|u-(u)|^2 \leq \rho^2\int_{B_\rho}|Du|^2 \leq C\rho^2\left(\frac{\rho}{R}\right)^n\int_{B_{\frac{R}{2}}}|Du|^2$

Poincaré

$\leq C\left(\frac{\rho}{R}\right)^{n+2}\displaystyle\int_{B_R}|u-(u)_{B_R}|^2$

Caccioppoli

⊥.

DR 5.12:

Najdeme P polynom řádu ℓ, tak, aby

$$\forall \text{ multiindex } \alpha \text{ řádu } \le \ell: \int_{B_\rho(0)} D^\alpha(u-P) = 0$$

$$\cancel{\int_{B_\rho(0)} |D^{\ell+1}(u-P)|^2 \le \int_B}$$

$$\int_{B_\rho} |u-P|^2 \underset{\text{Poincaré } B_\rho}{\le} \rho^2 \int |\nabla(u-P)|^2 \le \dots \le$$

$$\le C\rho^{2(\ell+1)} \int_{B_\rho} |\nabla^{\ell+1} u|^2 \underset{\rho<R}{\le} C\rho^{2(\ell+1)} \left(\frac{\rho}{R}\right)^n \int_{B_R} |\nabla^{\ell+1} u|^2$$

Caccioppoli

$$\le C\rho^{2(\ell+1)} \left(\frac{\rho}{R}\right)^n \left(\frac{1}{R}\right)^{2(\ell+1)} \int_{B_{2^{\ell+1}R}} |u|^2 \le$$

$$\le C\left(\frac{\rho}{R}\right)^{n+2(\ell+1)} R^n R^{2\ell} \xrightarrow{R\to+\infty} 0$$

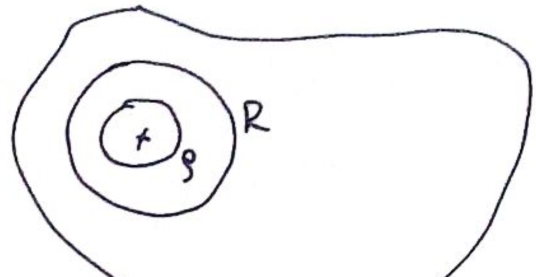
jako $\frac{1}{R^2}$

$R>1$

$u = P$ v $\mathbb{R}^n$ $\perp$

$$\Rightarrow u = P \text{ na } B_\rho \Rightarrow \nabla^{\ell+1} u = 0 \text{ v } B_\rho \Rightarrow \nabla^{\ell+1} u = 0 \text{ v } \mathbb{R}^n$$

DŘT 5.14:



$B_\varrho \subset B_R \subset 2B_R \subset \Omega$

$$\int_{B_\varrho} |Du - (Du)_\varrho|^2 \overset{\pm Dv \,\pm (Dv)_\varrho}{\le} c \int_{B_\varrho} \left(|Dv - (Dv)_\varrho|^2 + |Du - Dv|^2 \right) = \mathrm{I} + \mathrm{II}$$

Pomocí fce v.

$$\int_{B_\varrho} |(Du)_\varrho - (Dv)_\varrho|^2 = \cancel{\int_{B_\varrho} \Big| \int_{B_\varrho} \fint_{B_\varrho} u(x) - u(y)} = \int_{B_\varrho} \left| \fint_{B_\varrho} Du(x) - Dv(x)\, dx \right|^2$$

$$\le \int_{B_\varrho} \fint_{B_\varrho} |Du(x) - Dv(x)|^2 dx = \int_{B_\varrho} |Du(x) - Dv(x)|^2 dx$$

Volba v: Řešení rce: $D_\alpha (A^{\alpha\beta}_{ij} D_\beta v^j) = 0$ v B_R

$v = u$ na ∂B_R

$$\mathrm{II} + \mathrm{I} \le c \left(\frac{\varrho}{R}\right)^{n+2} \int_{B_R} |Dv - (Dv)_R|^2 \le c \left(\frac{\varrho}{R}\right)^{n+2} \int_{B_R} |Du - (Du)_R|^2$$

$$+ c \int_{B_R} |Du - Dv|^2$$

Odhad $\int_{B_R} |D(u-v)|^2$: Odečteme rce pro u a v a testujeme

$w = u - v$

$$\int_{B_R} A_{ij}^{\alpha\beta} D_\beta(u^j - v^j)\, D_\alpha(u^i - v^i) = -\int_{B_R} \underset{(-\,\mathrm{konst})}{F_i^\alpha}\, D_\alpha(u^i - v^i)$$

$$\int_{B_R} |\nabla(u-v)|^2 \le c\int_{B_R} |F - (F)_R|^2 \le c R^{n} [F]^2_{\mathcal{L}^{2,n}}$$

Dokonaly:

$$\underbrace{\int_{B_\rho} |\nabla u - (\nabla u)_\rho|^2}_{\phi(\rho)} \le c\left(\left(\frac{\rho}{R}\right)^{\overbrace{n+2}^{\alpha}} \underbrace{\int_{B_R} |\nabla u - (\nabla u)_R|^2}_{\phi(R)} + R^{n} \underbrace{[F]^2_{\mathcal{L}^{2,n}}}_{B}\right)$$

Lemma a další závěr.

5.13

$$\Rightarrow \int_{B_\rho} |\nabla u - (\nabla u)_\rho|^2 \le C\left(\frac{1}{R^n}\int_{B_R} |\nabla u - (\nabla u)_B|^2 + c\,[F]^2_{\mathcal{L}^{2,n}}\right)\rho^{n}$$

$\perp$.