mmma 583 — 2.11.

ϱ volíme jako aR: Víme $\alpha > \beta$

Vol $\alpha > \gamma > \beta$

$$A\left(\frac{\varrho}{R}\right)^{\alpha} = \left(\frac{\varrho}{R}\right)^{\gamma}\left(\left(\frac{\varrho}{R}\right)^{\alpha-\gamma} A\right)$$

malé $\leq \frac{1}{2}$

$\varrho := aR$ $\quad a\in(0,1)$

$$A\left(\frac{\varrho}{R}\right)^{\alpha} = a^{\gamma}\left(a^{\alpha-\gamma}A\right) \longrightarrow$$ Volím A tak malé aby $a^{\alpha-\gamma}A \leq \frac{1}{2}$

chci $A\varepsilon \leq \frac{a^{\gamma}}{2}$ Volba $\varepsilon_0 > 0$.

Pro $\varrho = aR$: $\phi(aR) \leq a^{\gamma}\phi(R) + BR^{\beta}$

$$\phi(a^2R) \leq a^{\gamma}\phi(aR) + B(aR)^{\beta}$$
$$\leq a^{2\gamma}\phi(R) + BR^{\beta}(a^{\gamma} + a^{\beta})$$

$$\phi(a^3R) \leq a^{\gamma}\phi(a^2R) + B(a^2R)^{\beta}$$
$$\leq a^{3\gamma}\phi(R) + BR^{\beta}(a^{2\gamma} + a^{\gamma+\beta} + a^{2\beta})$$

$$\phi(a^{k}R) \leq a^{k\gamma}\phi(R) + BR^{\beta}\underbrace{\left(a^{(k-1)\gamma} + \dots + a^{j\gamma+((k-1)-j)\beta} + \dots + a^{(k-1)\beta}\right)}_{\sum_{j=0}^{k-1} a^{j\gamma+((k-1)-j)\beta}}$$

$$\leq a^{k\gamma}\phi(R) + BR^{\beta}a^{(k-1)\beta}\sum_{j=0}^{k-1} a^{j(\gamma-\beta)}$$

$k \in \mathbb{N}$:

$$\phi(a^k R) \le a^{k\gamma}\phi(R) + BR^\beta a^{(k-1)\beta}\frac{1}{1-a^{\gamma-\beta}}$$

Vezmi $0<\rho<R$

$$\rho \in (a^{k+1}R;\ a^k R]$$

$$\phi(\rho) \le \phi(a^k R) \le \underbrace{a^{k\gamma}}_{\left(\frac{a^{(k+1)}R}{R}\right)^\gamma \cdot \left(\frac{1}{R}\right)^\gamma \cdot a^{-\gamma}}\phi(R) + B\underbrace{R^\beta a^{(k-1)\beta}}_{(a^{k+1}R)^\beta \cdot a^{-2\beta}}\frac{1}{1-a^{\gamma-\beta}}$$

$$\le \left(\frac{\rho}{R}\right)^\gamma a^{-\gamma}\phi(R) + B\rho^\beta \frac{a^{-2\beta}}{1-a^{\gamma-\beta}}$$

$$\le \rho^\beta\left(\frac{\phi(R)}{R^\beta} + B\right)\cdot C(a)$$

$\perp$.

Dk: T 5.17:

$B_\rho \subset B_R \subset \Omega$ se středy $x_0 \in \Omega$

v B_R: $\quad D_\alpha(A_{ij}^{\alpha\beta}(x_0)\, D_\beta u^i) = D_\alpha([A_{ij}^{\alpha\beta}(x_0) - A_{ij}^{\alpha\beta}]\, D_\beta u^i)$

$$- D_\alpha F_\alpha^i$$

Jako v minulé větě:

$$\int_{B_\rho}|\nabla u|^2 \le c\left(\frac{\rho}{R}\right)^n\int_{B_R}|\nabla u|^2 + c\int_{B_R}|F|^2 + c\int_{B_R}|\nabla u|^2\,\omega(R)^2$$

$$\underbrace{\int_{B_\varrho} |Du|^2}_{\phi(\varrho)} \le c\left(\left(\frac{\varrho}{R}\right)^n + \underbrace{\omega(R)^2}_{\varepsilon}\right)\underbrace{\int_{B_R} |Du|^2}_{\phi(R)} + cR^\lambda \|F\|^2_{L^{2,\lambda}(B_R)}$$

Lemma: $\exists \varepsilon_0 > 0$: Pokud $\omega(R)^2 < \varepsilon_0$

$$\Rightarrow \quad (*) \quad \int_{B_\varrho} |Du|^2 \le \varrho^\lambda \left(\left(\frac{1}{R}\right)^\lambda \int_{B_R} |Du|^2 + \|F\|^2_{L^{2,\lambda}(B_R)} \right) C$$

Jak udělat $\omega(R)$ malé? Z A je st. spoj. na kompaktech v Ω.

Krok 2: Fix $K \subset\subset \tilde{\Omega}$ kompakt.

$\Rightarrow \exists R_0 > 0 : \mathcal{U}(K, 2R_0) \subset \tilde{\Omega}$

$\Rightarrow A$ je st. spojité na $\overline{\mathcal{U}(K, R_0)}$.

$\Rightarrow$ Pro $\varepsilon_0 > 0$ z lemmatu ex. $R_1 > 0$, $\forall\, 0 < R < R_1 < \frac{R_0}{2}$

Platí $(*)$ s $x_0 \in \mathcal{U}(K, \frac{R_0}{2})$

$\Rightarrow$ Tvrzení T 5.17 (5.26). $\perp$.

Pro $R > R_1$: $\varrho \ge \frac{R_1}{2}$ hiv

$\varrho < \frac{R_1}{2}$ použij $(*)$ s ϱ a $\frac{R_1}{2}$ a pak hiv.

DR T5.19:

Krok 1: $\nabla u \in L^{2,\lambda}_{loc}(\Omega)$ $\forall \lambda \in (0,n)$

Krok 2: suboptimální odhad

$B_\rho \subset B_R \subset \Omega$: zamrazí koef. v x_0 střed koule

$$D_\alpha\left(A^{\alpha\beta}_{ij}(x_0)\, D_\beta u^j\right) = D_\alpha\left(\left[A^{\alpha\beta}_{ij}(x_0) - A\right] D_\beta u^j\right) - D_\alpha F^i_\alpha$$

Z dk. V 5.14:

$$(**)\quad \int_{B_\rho} |\nabla u - (\nabla u)_\rho|^2 \le C\left(\left(\frac{\rho}{R}\right)^{n+2} \int_{B_R} |\nabla u - (\nabla u)_R|^2 + R^\lambda [F]^2_{\mathcal{L}^{2,\lambda}} + [A]^2_{C^{0,\delta}} R^{2\delta} \underbrace{\int_{B_R} |\nabla u|^2}_{\|\nabla u\|^2_{L^{2,n-\varepsilon}} R^{n-\varepsilon}}\right)$$

Lemma 1 $\Rightarrow$ $\nabla u \in \mathcal{L}^{2,n-\varepsilon+2\delta}$ kde $n-\varepsilon+2\delta \in (n,\lambda)$

$$(**):\quad \int_{B_\rho} |\nabla u - (\nabla u)_\rho|^2 \le C\left(\left(\frac{\rho}{R}\right)^{n+2} \int_{B_R} |\nabla u - (\nabla u)_R|^2 + R^\lambda [F]^2_{\mathcal{L}^{2,\lambda}} + [A]^2_{C^{0,\delta}} R^{2\delta} \|\nabla u\|^2_{L^\infty(B_R)} \cdot R^n\right)$$

$C^{0,\delta} = \mathcal{L}^{2,n+2\delta}$ $\quad \lambda = n+2\delta$

Lemma 1 $\Longrightarrow$ $\nabla u \in \mathcal{L}^{2,n+2\delta}_{loc}(\Omega) = C^{0,\alpha}_{loc}(\Omega)$