mmma 583 — 16.11.

$\operatorname{div}(A(u), \nabla u) = \dots$

Protipříklady: De Giorgi, Giusti & Miranda

V příkladu "De Giorgi" musíme mít $\alpha = 1$

$u(x) = \frac{x}{|x|}$ nespojité pro n ≥ 3

$$\operatorname{div}\left(A\left(1, \tfrac{2}{n-2}, x\right) \nabla u\right) = 0$$

$$A\left(1, \tfrac{2}{n-2}, x\right) = \tilde{A}(u)$$

Pozn.: $\frac{x_i x_\gamma}{|x|^2} = u_i u_\gamma$ ... nezávislé na u

$$\frac{x_i x_\gamma}{|x|^2} = \frac{u_i u_\gamma}{1+1} \cdot 2 = \frac{u_i u_\gamma}{1+|u|^2} \cdot 2$$

Dk T. 4.23:

Fix $\alpha \in (0,1)$, $x_0 \in Reg_0(u)$ ... Chceme $u \in C^{0,\alpha}(\mathcal{U}(x_0))$.

Volba $\tau > 0$, zlomení c_*: $\tau^{2(1-\alpha)} c_* < 1$.

[Pozn.: (4.17): $E(x_0, \tau R) \le \tau^{2\alpha} E(x_0, R)$

$$E(x_0,\varrho) = \fint_{B(x_0,\varrho)} |u - (u)_{x_0,\varrho}|^2$$

Ex. ε_0, R_0 z L 4.21.

Vol $R \in (0, R_0)$ tak, aby $B_R(x_0) \subset \Omega$ a

$E(x_0, R) < \varepsilon_0^2$ (lze $x_0 \in Reg_0(u)$)

$\Rightarrow$

$$\underline{E(x_0, \tau R)} \le \tau^{2\alpha} E(x_0, R) < \underline{\varepsilon_0^2}$$] s $y \in \mathcal{U}(x_0, \delta)$

$\forall k \in \mathbb{N}$: $E(x_0, \tau^k R) \le \tau^{2\alpha k} (E(x_0, R))$

$\Rightarrow$ $\varrho \in (0, R)$: Najdu $k \in \mathbb{N}$, $\varrho \in (\tau^k R, \tau^{k-1} R]$

$$E(x_0, \varrho) \le c\, E(x_0, \tau^{k-1} R) \le c\, E(x_0, R)\, \tau^{2\alpha(k-1)}$$

$$\le c\, E(x_0, R) (\tau^k R)^{2\alpha} \cdot \frac{\tau^{-2\alpha}}{R^{2\alpha}} \le c\, E(x_0, R) \left(\frac{\varrho}{R}\right)^{2\alpha} \cdot \tau^{-2\alpha}$$

$$\le c(\tau, R, \ldots)\, \varrho^{2\alpha}$$

Tento odhad chceme mít silný: $\exists \delta > 0 : \forall y \in \mathcal{U}(x_0, \delta)$:
$E(y, R) < \varepsilon_0^2$.

$\Rightarrow u \in C^{0,\alpha}(\overline{\mathcal{U}(x_0, \delta)})$

Dk: Prop. 9.21:

$$\Sigma_\alpha = \bigcup_{s>0} \overbrace{\left\{ x \in \Omega ;\ \limsup_{\varrho \to 0+} \frac{1}{\varrho^\alpha} \int_{B_\varrho(x)} |f| > \frac{1}{s} \right\}}^{\Sigma_{\alpha,s}}$$

Chci: $\mathcal{H}^\alpha(\Sigma_{\alpha,s}) = 0$.

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$\mathcal{H}^\alpha_\delta$; fix $\delta > 0$.

$$\forall x \in \Sigma_{\alpha,s},\ \exists \varrho_x \in (0,\delta) : \frac{1}{\varrho_x^\alpha} \int_{B_{\varrho_x}(x)} |f| > \frac{1}{s}$$

$\left\{ \overline{B_{\varrho_x}(x)} \right\}_{x \in \Sigma_{\alpha,s}}$ pokrytí ~~...~~ neostré $\Sigma_{\alpha,s}$

Vitali

$\Rightarrow \exists$ spočetné podpokrytí, po 2 disj. $\{B_i\}$

a $\bigcup 5\overline{B}_i \supset \Sigma_{\alpha,s}$, B_i má pol. ϱ_i

$$\mathcal{H}^\alpha_\delta(\Sigma_{\alpha,s}) \leq 5^\alpha \sum_{i=1} \omega_\alpha \varrho_i^\alpha \leq C s \sum_i \int_{B_{\varrho_i}} |f| = C \int_{\bigcup B_{\varrho_i}} |f|$$

Ukážeme, že $\lambda^n(\bigcup B_i)$ je malá!

$$\lambda^n(\bigcup B_i) = c \sum_i \varrho_i^n \leq c\,\delta^{n-\alpha} \sum_i \varrho_i^\alpha \leq c\,\overbrace{\delta^{n-\alpha}}^{>0} \int_\Omega |f|$$

$$\Rightarrow \mathcal{H}^\alpha_\delta(\Sigma_{\alpha,s}) \xrightarrow{\delta \to 0+} 0 \Rightarrow \mathcal{H}^\alpha(\Sigma_{\alpha,s}) = 0$$

$\perp$

Dk Corollary 4.25:

$$\liminf_{\varrho \to 0+} \fint_{\Omega(x_0,\varrho)} |u-(u)|^2 \leq \limsup \fint_{\Omega(x_0,\varrho)} |u-(u)|^2$$

$$\leq \limsup_{\varrho \to 0+} \underbrace{\varrho^2 \fint_{\Omega(x_0,\varrho)} |\nabla u|^2}_{\frac{1}{\varrho^{n-2}} \int_{B_\varrho(x_0)} |f|}$$

$; \quad f := |\nabla u|^2$

ϱ malé

$$\Rightarrow \quad \mathrm{Sing}_0(u) \subset \Sigma_{n-2}(|\nabla u|^2)$$

$$\overset{\text{P.9.21}}{\Rightarrow} \quad \mathcal{H}^{n-2}(\mathrm{Sing}_0(u)) = 0.$$

$\perp$.