DR L 4.21: 2. verse

Fix $\tau > 0$, $R > 0$ tak, aby $B_R(x_0) = B_R \subset \Omega$.
$x_0 \in \Omega$

Dále předpokládejme na $B_{R/2}$, u je slabé řešení.

Pp. $\tilde{E}(u, R/2) > 0$. (jinak triv.)

Def: $v := \dfrac{u}{\tilde{E}(u, R/2)^{1/2}}$, kde $\tilde{E}(u, R/2) := \left(\frac{R}{2}\right)^2 \fint_{B_{R/2}} |\nabla u|^2$

Pak $\displaystyle \fint_{B_{R/2}} |\nabla v|^2 = \frac{\fint_{B_{R/2}} |\nabla u|^2}{\left(\frac{R}{2}\right)^2 \fint_{B_{R/2}} |\nabla u|^2} = \left(\frac{2}{R}\right)^2$

$$\Rightarrow \left(\frac{R}{2}\right)^2 \fint_{B_{R/2}} |\nabla v|^2 \leq 1, \quad \rho = \frac{R}{2}; \ \gamma = 1$$

$$\left| \left(\frac{R}{2}\right) \fint_{B_{R/2}} A(x_0, (u)_{R/2}) \nabla v \cdot \nabla \varphi \right| \leq$$

$$c(n, L)\, \omega^{1/2}\left(\frac{R}{2} + \left(\fint_{B_{R/2}} |u - (u)_{R/2}|^2\right)^{1/2}\right) \sup_{B_{R/2}} |\nabla \varphi|$$

Pokud malé

$$\Rightarrow h \in W^{1,2}(B_{R/2}): \fint_{B_{R/2}} |v - h|^2 \leq \varepsilon, \quad \fint_{B_{R/2}} |\nabla h|^2 \left(\frac{R}{2}\right)^2 \leq 1$$

$E(u,\tau R) = \fint_{B_{\tau R}} |u-(u)_{\tau R}|^2 \le \fint_{B_{\tau R}} |u - \tilde{E}(u,R/2)^{1/2}(h)_{\tau R}|^2$

Pp. $\tau < \frac{1}{2}$

$\sim \tilde{E}(u)^{1/2} w$

$$\le c\left(\fint_{B_{\tau R}} \tilde{E}(u,R/2)|w-h|^2 + \tilde{E}(u,R/2)\fint_{B_{\tau R}}|h-(h)_{\tau R}|^2 \right.$$

$$\le c\cdot\tilde{E}(u,R/2)\left((2\tau)^{-n}\fint_{B_{R/2}}|w-h|^2 + (2\tau)^2\fint_{B_{R/2}}|h-(h)_{R/2}|^2\right)$$

$(w)_{R/2}$

$\pm w$

$$\le c\,\tilde{E}(u,R/2)\left((2\tau)^{-n}\fint_{B_{R/2}}|w-h|^2 + (2\tau)^2\fint_{B_{R/2}}|w-(w)_{R/2}|^2\right)$$

$$\le c\fint_{B_{R/2}}|u-(u)_{R/2}|^2\tau^2 + \left(\tilde{E}(u,R/2)(2\tau)^{-n}\varepsilon\right)$$

$\left(\frac{R}{2}\right)^2\fint_{B_{R/2}}|Du|^2 \le c\fint_{B_R}|u-(u)_R|^2$ Caccioppoli

$$\le c\fint_{B_R}|u-(u)_R|^2\left(\tau^2+\tau^{-n}\varepsilon\right) = cE(u,R)\left(\tau^2+\tau^{-n}\varepsilon\right)$$

a) $\tau\in[1/2,1)$ hiv!

b) $\tau\in(0,\frac{1}{2}) \longrightarrow$ Vál $\varepsilon>0$: $\tau^{-n}\varepsilon<\tau^2 \longrightarrow \delta>0$ a L 4.27

$\longrightarrow R_0>0,\ \varepsilon_0>0.$

DR L 4.21: Verze 3 - přímá

ať $B_R \subset \Omega$.

Pomocný problém: $\operatorname{div}(a(x_0,(u)_{R/4})\nabla v) = 0 \quad B_{R/4}$

$v = u \quad \partial B_{R/4}$

Blízkost k u: $\operatorname{div}(a(x,u)\nabla u) = 0$ v $B_{R/4}$

$u = u$ na $\partial B_{R/4}$

$$\operatorname{div}\left(a(x_0,(u)_{R/4})(\nabla v - \nabla u)\right) = \operatorname{div}\left(\left(a(x,u) - a(x_0,(u)_{R/4})\right)\nabla u\right)$$

/ test $v-u$, $\int_{B_{R/4}}$

$$\fint_{B_{R/4}} |\nabla v - \nabla u|^2 \le C \fint_{B_{R/4}} |a(x,u) - a(x_0,(u)_{R/4})|^2 |\nabla u|^2 = *$$

$$\boxed{P_P: \left(\fint_{B_{R/4}} |\nabla u|^q\right)^{1/q} \le \left(\fint_{B_{R/2}} |\nabla u|^2\right)^{1/2} \text{ pro nějaké } q > 2}$$

$$(*) \le C\left(\fint_{B_{R/4}} |\nabla u|^q\right)^{2/q} \left(\fint_{B_{R/4}} |a(x,u) - a(x_0,(u)_{R/4})|^{\frac{4}{q-2}}\right)^{\frac{q-2}{2}}$$

$$1 - \frac{2}{q} = \frac{q-2}{2}$$

$$\le C(L) \fint_{B_{R/2}} |\nabla u|^2 \left(\fint_{B_{R/4}} \omega\left(|x-x_0| + |u-(u)_{R/4}|\right)\right)^{\frac{q-2}{2}}$$

$$\le C \fint_{B_{R/2}} |\nabla u|^2 \quad \omega^{\frac{q-2}{2}}\left(\frac{R}{4} + \left(\fint_{B_{R/4}} |u-(u)_{R/4}|^2\right)^{1/2}\right)$$

$\tau < \frac{1}{4}$

$$E(u,\tau R) \underset{\text{Poincaré}}{\leq} c\tilde{E}(u,\tau R) = c \fint_{B_{\tau R}} |\nabla u|^2 (\tau R)^2$$

$$\leq c(\tau R)^2 \left(\fint_{B_{\tau R}} |\nabla(u-v)|^2 + \fint_{B_{\tau R}} |\nabla v|^2 \right)$$

$$\leq c(\tau R)^2 \left(\omega^{\frac{q-2}{2}} (\dots) \tau^{-n} + 1 \right) \fint_{B_{R/2}} |\nabla u|^2$$

Caccipolli

$$\leq c\tau^2 \underbrace{\fint_{B_R} |u-(u)_R|^2}_{E(u,R)} \underbrace{\left(1+\omega^{\frac{q-2}{2}} \left(\frac{R}{4} + \left(\fint_{B_{R/2}} |u-(u)_{R/4}|^2 \right)^{1/2} \right) \tau^{-n} \right)}_{\text{je omezené?}}$$

Ano: Podm. na R_0 a ε_0.

$\perp$