An $N \times N$ *Toeplitz matrix* is a matrix whose entries are constant along diagonals:

$$\mathbf{A} = \begin{pmatrix} a_0 & a_{-1} & & \cdots & a_{1-N} \\ a_1 & a_0 & & & \vdots \\ & \ddots & \ddots & \ddots & \\ \vdots & & & a_0 & a_{-1} \\ a_{N-1} & \cdots & & a_1 & a_0 \end{pmatrix}. \tag{7.1}$$

A semi-infinite matrix of the same form is known as a *Toeplitz operator*, and a doubly infinite matrix of this kind is a *Laurent operator*.[1] A *circulant* matrix, which is the finite-dimensional analogue of a Laurent operator, is a special case of a Toeplitz matrix in which the entries wrap around periodically: $a_j = a_{j-N}$ for $1 \le j \le N-1$.

The *symbol* of a Toeplitz matrix or Toeplitz operator or Laurent operator is the function

$$f(z) = \sum_k a_k z^k\,; \tag{7.2}$$

Spectra of Toeplitz and Laurent operators
Theorem 7.1 *Let* $\mathbf{A}$ *be a circulant matrix or Laurent or Toeplitz operator with continuous symbol* f.   (i) *If* $\mathbf{A}$ *is a circulant matrix, then* $\sigma(\mathbf{A}) = f(\mathbb{T}_N)$.   (ii) *If* $\mathbf{A}$ *is a Laurent operator, then* $\sigma(\mathbf{A}) = f(\mathbb{T})$.   (iii) *If* $\mathbf{A}$ *is a Toeplitz operator, then* $\sigma(\mathbf{A})$ *is equal to* $f(\mathbb{T})$ *together with all the points enclosed by this curve with nonzero winding number.*

In §7 we considered four types of matrices:

	no boundary	*boundary*
infinite	Laurent operator on $\{\ldots,-2,-1,0,1,2,\ldots\}$	Toeplitz operator on $\{1,2,3,\ldots\}$
finite	circulant matrix on $\{1,2,\ldots,N\}$ (periodic)	Toeplitz matrix on $\{1,2,\ldots,N\}$ (nonperiodic)

Nyní:

Let $a_0, \ldots, a_d$ ($a_d \neq 0$) be a set of real or complex numbers, and let $\mathbf{A}$ denote the degree-d differential operator

$$a_0 + a_1 \frac{\mathrm{d}}{\mathrm{d}x} + \cdots + a_d \frac{\mathrm{d}^d}{\mathrm{d}x^d} \tag{10.1}$$

acting in L^2 on a domain and with boundary conditions to be specified.[1] The *symbol* of (10.1) is the function

$$f(k) = \sum_{j=0}^{d} a_j (-\mathrm{i}k)^j, \qquad k \in \mathbb{R}. \tag{10.2}$$

	no boundary	*boundary*
infinite	constant-coefficient differential operator on $(-\infty, \infty)$	constant-coefficient differential operator on $[0, \infty)$
finite	constant-coefficient differential operator on $[0, L]$ (periodic)	constant-coefficient differential operator on $[0, L]$ (nonperiodic)

Spectra of constant-coefficient differential operators

Theorem 10.1 *Let $\mathbf{A}$ be a degree-d constant-coefficient differential operator with symbol f: on $[0, L]$ with periodic boundary conditions, on $[0, \infty)$ with β homogeneous boundary conditions at $x = 0$ ($0 \leq \beta \leq d$), or on $(-\infty, \infty)$.*

(i) *On $[0, L]$, $\sigma(\mathbf{A}) = f(2\pi\mathbb{Z}/L)$.*

(ii) *On $(-\infty, \infty)$, $\sigma(\mathbf{A}) = f(\mathbb{R})$.*

(iii) *On $[0, \infty)$, $\sigma(\mathbf{A})$ is equal to $f(\mathbb{R})$ together with all the points enclosed by this curve with winding number that differs from $d - \beta$.*

In this theorem, the statement that there are β homogeneous boundary conditions means that

$$u(0) = u'(0) = \cdots = u^{(\beta-1)}(0) = 0.$$

• Operátory na $(-\infty;\infty)$: $A: \mathcal{D}(A) \to L^2(\mathbb{R})$

$$u \longmapsto a_0 u + a_1 u' + \dots + a_d u^{(d)}$$

$$\mathcal{D}(A) = W^{d,2}(\mathbb{R}) = \{ g \in L^2(\mathbb{R}) : \forall j \in \{0; \dots; d\} \text{ je } \underbrace{g^{(j)}}_{\text{slabá derivace}} \in L^2(\mathbb{R}) \}$$

Potom je pro Au definovaná Plancherelova (rozšířená Fourierova) transformace a platí $\widehat{Au}(k) = \widehat{a_0 u + a_1 u' + \dots + a_d u^{(d)}}(k) =$

$$= a_0 \hat{u}(k) - (ik)\hat{u}(k) + (ik)^2 \hat{u}(k) + \dots + (-1)^d (ik)^d \hat{u}(k) = f(k)\hat{u}(k).$$

Definice na FA: $\hat{u}(x) = \int_{-\infty}^{\infty} u(t) e^{-ixt} \, dt \cdot \text{konstanta} \overset{FA}{\Rightarrow} \widehat{u'}(x) = (ix)\hat{u}(x)$

Definice zde: $\hat{u}(x) = \int_{-\infty}^{\infty} u(t) e^{ixt} \, dt \cdot \text{konstanta} \Rightarrow \widehat{u'}(x) = (-ix)\hat{u}(x)$
(nejspíš)

Definujme $\hat{A} : \mathcal{D}(\hat{A}) \subset L^2(\mathbb{R}) \to L^2(\mathbb{R})$

$$\hat{u} \longmapsto f(k)\hat{u}(k)$$

$$\mathcal{D}(\hat{A}) = \{ \hat{u} \in L^2(\mathbb{R}) : |k^d \hat{u}(k)| \in L^2(\mathbb{R}) \}$$

Potom $\sigma(\hat{A}) = \overline{f(\mathbb{R})} = f(\mathbb{R})$ (uzávěr) $\left(\begin{array}{l} (\lambda - \hat{A})\hat{u} = \hat{v} \Leftrightarrow (\lambda - f)\hat{u} = \hat{v} \\ \hat{u} = \dfrac{\hat{v}}{\lambda - f} \end{array} \right)$

$$u \longmapsto a_0 u + a_1 u' + \dots + a_d u^{(d)}$$

$$A : \mathcal{D}(A) \subset L^2(\mathbb{R}) \longrightarrow L^2(\mathbb{R})$$

$$\updownarrow \mathcal{F} \qquad\qquad \updownarrow \mathcal{F} \text{ ... tam i zpátky spojitá bijekce}$$

$$\hat{A} : \mathcal{D}(\hat{A}) \subset L^2(\mathbb{R}) \longrightarrow L^2(\mathbb{R})$$

$$\hat{u} \longmapsto f(k)\hat{u}(k) = a_0 \hat{u}(k) - (ik)\hat{u}(k) + \dots + (-1)^d (ik)^d \hat{u}(k)$$

$$\sigma(A) = \sigma(\hat{A}) = f(\mathbb{R})$$

• operátory na $[0;L]$ s periodickými okrajovými podmínkami (pro $L=2\pi$):

$$A : D(A) \subset L^2(0;2\pi) \longrightarrow L^2(0;2\pi)$$

$$D(A) = \{ g \in W^{d,2}(0;2\pi) : g(0) = g(2\pi); \dots; g^{(d)}(0) = g^{(d)}(2\pi) \} ?$$

$$\mathcal{F} : u \longmapsto \hat{u} = (a_k)_{k=-\infty}^{\infty} \in \ell^2(\mathbb{Z}), \ a_k = \text{konstanta} \cdot \int_0^{2\pi} u(t) e^{ikt}\, dt$$

$$a_k(u') = c \cdot \int_0^{2\pi} u'(t) e^{ikt}\, dt = \underbrace{\left[c u(t) e^{ikt} \right]_0^{2\pi}}_{=0} - c \int_0^{2\pi} u(t)\, ik\, e^{ikt}\, dt = -ik\, a_k(u)$$

Tedy $(\widehat{Au})_k = f(k) (\hat{u})_k$, $f(k) = \sum_{j=0}^{d} a_j (-ik)^j$ ... opět symbol-funkce

Definujme $\hat{A} : D(\hat{A}) \subset \ell^2(\mathbb{Z}) \longrightarrow \ell^2(\mathbb{Z})$

$$(\hat{u})_k \longmapsto f(k) (\hat{u})_k \Rightarrow \sigma(\hat{A}) = f(\mathbb{Z})$$

$$D(\hat{A}) = \{ (a_k) \in \ell^2(\mathbb{Z}) : (|k|^d a_k) \in \ell^2(\mathbb{Z}) \}$$

$$u \longmapsto a_0 u + a_1 u' + \dots + a_d u^{(d)}$$

$$\begin{array}{ccc} A : D(A) \subset L^2(\mathbb{R}) & \longrightarrow & L^2(\mathbb{R}) \\ \updownarrow \mathcal{F} & & \updownarrow \mathcal{F} \\ \hat{A} : D(\hat{A}) \subset \ell^2(\mathbb{Z}) & \longrightarrow & \ell^2(\mathbb{Z}) \end{array}$$

$$(\hat{u})_k \longmapsto f(k) (\hat{u})_k, \ \sigma(A) = \sigma(\hat{A}) = f(\mathbb{Z})$$

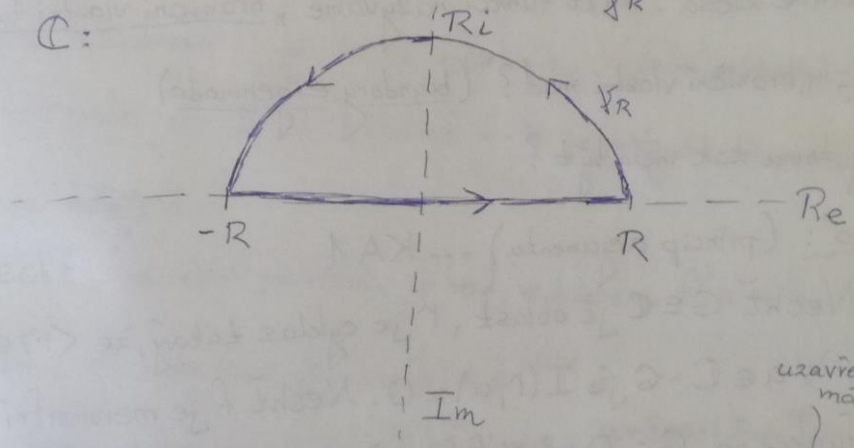
• Index bodu $\lambda \in \mathbb{C}$ vzhledem k „symbolové křivce" $f(\mathbb{R})$:

Je-li φ uzavřená cesta v $\mathbb{C}$, pak pro $\lambda \in \mathbb{C} \setminus \varphi([a,b])$ definujeme:

$$\left(\begin{array}{c} \varphi : [a,b] \to \mathbb{C},\ \varphi(a) = \varphi(b) \\ \varphi \text{ je po částech } C^1 \end{array} \right)$$

$$I(\varphi, \lambda) \equiv \frac{1}{2\pi i} \int_\varphi \frac{1}{z-\lambda}\, dz = \frac{1}{2\pi i} \int_a^b \frac{\varphi'(t)}{\varphi(t)-\lambda}\, dt \ldots$$ kolikrát křivka oběhne bod φ proti směru hodinových ručiček

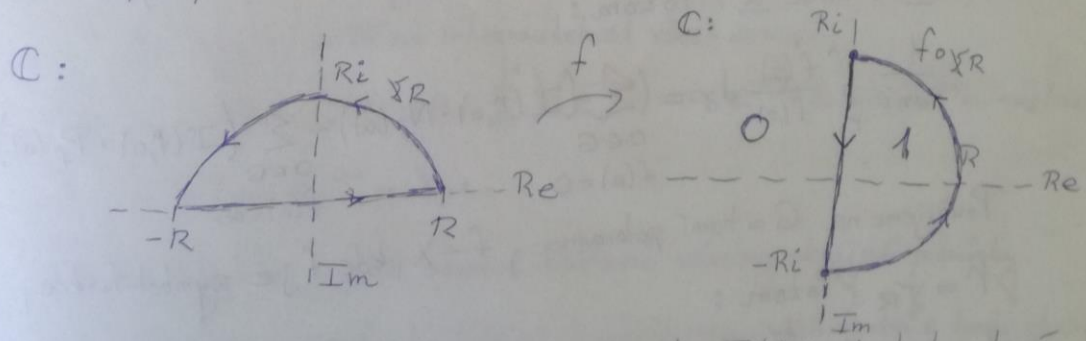
Pro symbolovou funkci $f(k) = \sum_{j=0}^{d} a_j (-ik)^j$ nemůžeme index definovat ihned, ale oklikou: Uvažme křivku γ_R



uzavřená cesta, má smysl

$I(f,\lambda)$ pro $\lambda \in \mathbb{C} \setminus f(\mathbb{R})$ definujeme jako $\lim_{R\to\infty} I(f \circ \gamma_R, \lambda)$.

Např. pro $Au = u'$ je $f(k) = -ik$



$\Rightarrow$ Všechny body v pravé polorovině mají index $I(f,\lambda) = 1$, body v levé polorovině mají index $I(f,\lambda) = 0$.

To znamená, že pokud uvažujeme operátor $Au = u'$ na $[0;\infty)$ bez okrajové podmínky v bodě $x=0$, tak ($d=1, \beta=0$) $G(A)$ = levá polorovina, neboť to je množina bodů, kde $I(f,\lambda) \neq d-\beta = 1$. Naopak pokud uvažujeme v $x=0$ okrajovou podmínku, tj. $\beta=1$ tak $G(A)$ = pravá polorovina, poněvadž to je množina všech bodů s indexem $I(f,\lambda) \neq d-\beta = 0$.

- $I(f,\lambda) < d-\beta$: Nejen že $\lambda \in G(A)$, ale dokonce existuje nenulová vlastní funkce u, jejíž absolutní hodnota pro $x \to \infty$ exponenciálně klesá. Tuto funkci nazýváme „hraniční vlastní funkce" nebo též „hraniční vlastní mód?" (boundary eigenmode)

Proč by tomu tak mělo být?

Věta: (princip argumentu) ... KA1

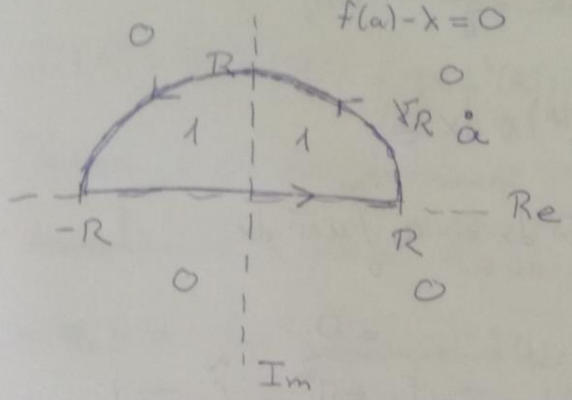
„Nechť $G \subseteq \mathbb{C}$ je oblast, Γ je cyklus takový, že $\langle\Gamma\rangle \subseteq G$ a $\forall a \in \mathbb{C} \setminus G$ je $I(\Gamma, a) = 0$. Nechť f je meromorfní funkce na G; $0, \infty \notin f(\langle\Gamma\rangle)$, $N_f(a)$ značí násobnost kořene funkce f v bodě a a $P_f(a)$ je násobnost pólu funkce f v bodě a. Potom:

$$\frac{1}{2\pi i}\int_\Gamma \frac{f'(z)}{f(z)}\,dz = \sum_{\substack{a\in G\\ f(a)=0}} \left(I(\Gamma,a)\cdot N_f(a)\right) - \sum_{\substack{a\in G\\ f(a)=\infty}} \left(I(\Gamma,a)\cdot P_f(a)\right)."$$

Použijeme na G = horní polorovina, $f-\lambda$, kde f je symbol-funkce, $\Gamma = \gamma_R$. Potom:

$$\underbrace{\frac{1}{2\pi i}\int_{f\circ\gamma_R} \frac{1}{z-\lambda}\,dz}_{= I(f\circ\gamma_R,\lambda)} = \frac{1}{2\pi i}\int_a^b \frac{f'(\gamma_R(t))}{f(\gamma_R(t))-\lambda}\gamma_R'(t)\,dt = \frac{1}{2\pi i}\int_{\gamma_R} \frac{f'(z)}{f(z)-\lambda}\,dz = \frac{1}{2\pi i}\int_{\gamma_R} \frac{(f(z)-\lambda)'}{f(z)-\lambda}\,dz$$

Tedy $I(f\circ\gamma_R,\lambda) = \sum\limits_{\substack{a\in \text{horní polorovina} \\ f(a)-\lambda=0}} I(\gamma_R,a)\, N_f(a)$ (*)



Pro dané $a\in$ horní polorovina existuje R_a takové, že $I(\gamma_R,a)=1$ $\forall R\geq R_a$.

Limitním přechodem v (*) $\lim\limits_{R\to\infty}$ dostaneme, že $I(f,\lambda)$ = počet bodů v horní polorovině (včetně násobností), pro které $f(a)=\lambda$. V dolní polorovině je tedy těchto bodů $d-I(f,\lambda)$, neboť $f-\lambda$ je polynom stupně d, a tak má d kořenů.

Nechť $a\in$ dolní polorovina je bod, ve kterém $f(a)=\lambda$.

Potom $Au=\lambda u$ pro $u(x)=e^{-iax}$:

$e^{-iax}\in L^2([0,\infty))$, neboť $\int_0^\infty |e^{-iax}|^2\,dx \overset{a=\mathrm{Re}\,a+i\,\mathrm{Im}\,a}{=} \int_0^\infty |e^{-xi\,\mathrm{Re}\,a}\cdot e^{(\mathrm{Im}\,a)x}|^2\,dx =$

$= \int_0^\infty |e^{(\mathrm{Im}\,a)x}|^2\,dx = \int_0^\infty e^{2(\mathrm{Im}\,a)x}\,dx = \left[\frac{1}{2(\mathrm{Im}\,a)}e^{2(\mathrm{Im}\,a)x}\right]_0^\infty = \frac{1}{2\,\mathrm{Im}\,a}$, neboť $\mathrm{Im}\,a<0$.

Analogicky ověříme integrovatelnost všech derivací.

$Au = \left(a_0 + a_1\frac{d}{dx}+\dots+a_d\frac{d^d}{dx^d}\right)(e^{-iax}) = a_0e^{-iax}+a_1(-ia)e^{-iax}+\dots+a_d(-ia)^d e^{-iax}$

$= f(a)e^{-iax} = \lambda u.$

Tato funkce ještě nemusí splňovat všechny okrajové podmínky. Jsme schopni nalézt $d-I(f,\lambda)$ lineárně nezávislých funkcí s touto vlastností a okrajových podmínek je β. Pokud $\beta< d-I(f,\lambda) \iff I(f,\lambda)<d-\beta$, tak jsme schopni najít nenulovou vlastní funkci s požadovanými vlastnostmi.

• $\underline{I(f,\lambda) > d-\beta:}$ $\quad A: \mathcal{D}(A) = \{g \in W^{d,2}([0;\infty)) : g(0) = \dots = g^{(\beta-1)}(0) = 0\} \to L^2([0;\infty))$

$$\mathcal{D}(A^*) = \{g \in W^{d,2}([0;\infty)) : g(0) = \dots = g^{(d-\beta-1)}(0)\}$$

$$\langle Au, v\rangle = \int_0^\infty (a_0 u + a_1 u' + \dots + a_d u^{(d)}) \overline{v} \, dx \circleddash$$

$$\int_0^\infty a_1 u' \overline{v} \, dx = a_1 [u \overline{v}]_0^\infty - a_1 \int_0^\infty u \overline{v'} \, dx = -a_1 \int_0^\infty u \overline{v'} \, dx$$

$$g \in W^{1,2}([0;\infty)) \Rightarrow \lim_{x\to\infty} g(x) = 0 \ ?$$

$$\int_0^\infty a_2 u'' \overline{v} \, dx = a_2 \underbrace{[u' \overline{v}]_0^\infty}_{=0} - a_2 \int_0^\infty u' \overline{v'} \, dx = -a_2 \overbrace{[u \overline{v'}]_0^\infty}^{=0} + a_2 \int_0^\infty u \overline{v''} \, dx$$

$$\int_0^\infty a_d u^{(d)} \overline{v} \, dx = a_d [u^{(d-1)} \overline{v}]_0^\infty - a_d \int_0^\infty u^{(d-1)} \overline{v'} \, dx = \dots$$

$$\circleddash \int_0^\infty u \left(a_0 \overline{v} - a_1 \overline{v'} + \dots + (-1)^d a_d \overline{v^{(d)}}\right) dx = \int_0^\infty u \, \overline{\left(\overline{a_0} v - \overline{a_1} v' + \dots + (-1)^d \overline{a_d} v^{(d)}\right)} \, dx$$

$$\Rightarrow A^* v = \overline{a_0} v - \overline{a_1} v' + \dots + (-1)^d \overline{a_d} v^{(d)}$$

komplexní sdružení

Pokud A je uzavřený a hustě definovaný, tak $\sigma(A) = \overline{\sigma(A^*)}$?

$$A^T v := a_0 v - a_1 v' + \dots + (-1)^d a_d v^{(d)}$$

A^T je také diferenciální operátor stupně d na $[0;\infty)$, má $(d-\beta)$ homogenních okrajových podmínek, je-li $f(k)$ symbol-funkce operátoru f, pak $f(-k)$ je symbol-funkce operátoru A^T. Index pro tento operátor je $\hat{I} = d - I$ a $\hat{\beta} = d - \beta \Rightarrow \left[\hat{I} < d - \hat{\beta} \Leftrightarrow d - I < d - (d-\beta) \Leftrightarrow I > d - \beta\right]$.

Tedy existuje vlastní funkce příslušná λ pro operátor $A^T \Rightarrow \lambda \in \sigma(A^T) \Rightarrow$
$\Rightarrow \lambda \in \sigma(A)$.

Příklad na Větu 10.1:

A defined by

$$\mathbf{A}u = \left(1 + \frac{\mathrm{d}}{\mathrm{d}x}\right)^3 u = u + 3u' + 3u'' + u''' \tag{10.3}$$

with symbol

$$f(k) = (1 - \mathrm{i}k)^3 = \mathrm{i}(k + \mathrm{i})^3. \tag{10.4}$$

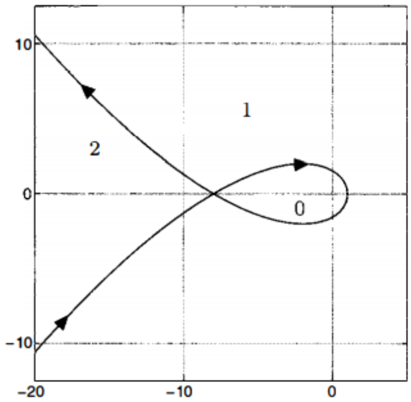


Figure 10.1: Symbol curve in the complex plane for the example (10.3)–(10.4). The numbers indicate regions associated with various winding numbers $I(f, \lambda)$.

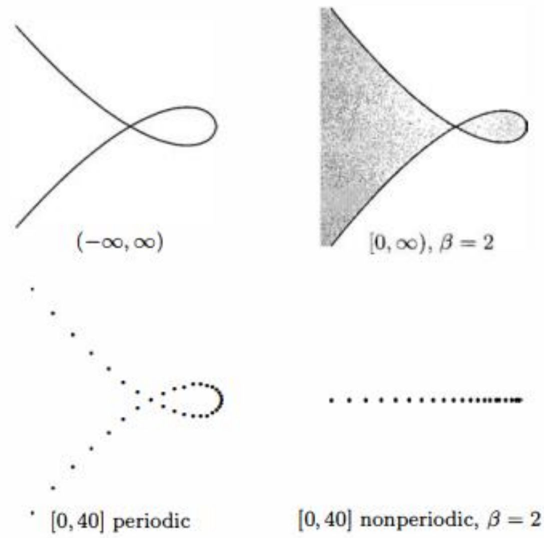


Figure 10.2: Rightmost parts of the spectra of constant-coefficient differential operators of the four types associated with the symbol (10.4). In the final case there are two boundary conditions at the left and one at the right.

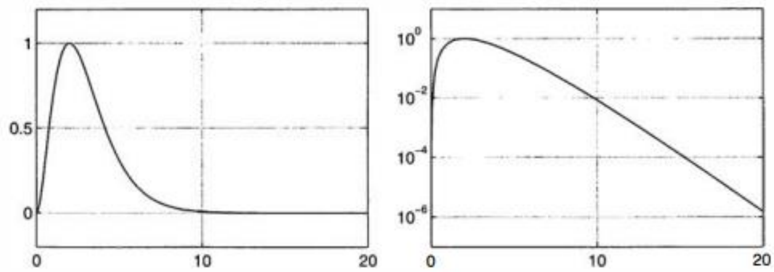


Figure 10.3: Eigenfunction $v(x) = x^2 e^{-x}$ of the differential operator (10.3) on $[0, \infty)$ with $\beta = 2$ associated with eigenvalue $\lambda = 0$ on a linear and a logarithmic scale. The eigenfunction is exponentially localized at the left boundary.

Pseudospectra of Toeplitz matrices

Theorem 7.2 *Let $\{\mathbf{A}_N\}$ be a family of banded or semibanded Toeplitz matrices as defined above, and let λ be any complex number with $I(f,\lambda) \neq 0$. Then for some $M > 1$ and all sufficiently large N,*

$$\|(\lambda - \mathbf{A}_N)^{-1}\| \geq M^N, \tag{7.10}$$

and there exist nonzero pseudoeigenvectors $\mathbf{v}^{(N)}$ satisfying

$$\frac{\|(\mathbf{A}_N - \lambda)\mathbf{v}^{(N)}\|}{\|\mathbf{v}^{(N)}\|} \leq M^{-N}$$

such that

$$\frac{|v_j^{(N)}|}{\max\limits_j |v_j^{(N)}|} \leq \begin{cases} M^{-j} & \text{if } I(f,\lambda) < 0, \\ M^{j-N} & \text{if } I(f,\lambda) > 0, \end{cases} \qquad 1 \leq j \leq N. \tag{7.11}$$

The constant M can be taken to be any number for which $f(z) \neq \lambda$ in the annulus $1 \leq |z| \leq M$ (if $I(f,\lambda) < 0$) or $M^{-1} \leq |z| \leq 1$ (if $I(f,\lambda) > 0$).

Pseudospectra of constant-coefficient differential operators

Theorem 10.2 *Let $\{\mathbf{A}_L\}$ be a family of degree-d constant-coefficient differential operators on $[0,L]$ with β homogeneous boundary conditions at $x = 0$ and γ homogeneous boundary conditions at $x = L$, and let λ be any complex number with $I(f,\lambda) < d - \beta$ or $I(f,\lambda) > \gamma$. Then for some $M > 0$ and all sufficiently large L,*

$$\|(\lambda - \mathbf{A}_L)^{-1}\| \geq \mathrm{e}^{LM}, \tag{10.9}$$

and there exist nonzero pseudomodes $v^{(L)}$ satisfying $\|(\mathbf{A}_L - \lambda)v^{(L)}\|/\|v^{(L)}\| \leq \mathrm{e}^{-LM}$ such that for all $x \in [0,L]$,

$$\frac{|v^{(L)}(x)|}{\sup\limits_x |v^{(L)}(x)|} \leq \begin{cases} \mathrm{e}^{-Mx} & \text{if } I(f,\lambda) < d - \beta; \\ \mathrm{e}^{-M(L-x)} & \text{if } I(f,\lambda) > \gamma. \end{cases} \tag{10.10}$$

The constant M can be taken to be any number for which $f(z) \neq \lambda$ in the strip $-M \leq \mathrm{Im} z \leq 0$ (if $I(f,\lambda) < d - \beta$) or $0 \leq \mathrm{Im} z \leq M$ (if $I(f,\lambda) > \gamma$).

Behavior of pseudospectra as $N \to \infty$

Theorem 7.3 *Let* $\mathbf{A}$ *be a Toeplitz operator with continuous symbol* f *and let* $\{\mathbf{A}_N\}$ *be the associated family of Toeplitz matrices. Then for any* $\varepsilon > 0$,

$$\lim_{N\to\infty} \sigma_\varepsilon(\mathbf{A}_N) = \sigma_\varepsilon(\mathbf{A}), \tag{7.18}$$

and thus

$$\lim_{\varepsilon\to 0}\lim_{N\to\infty} \sigma_\varepsilon(\mathbf{A}_N) = \sigma(\mathbf{A}). \tag{7.19}$$

Behavior of pseudospectra as $L \to \infty$

Theorem 10.3 *Let* $\mathbf{A}$ *be a degree-d constant-coefficient differential operator on* $[0,\infty)$ *with symbol* f *and* β *homogeneous boundary conditions at* $x = 0$ $(0 \le \beta \le d)$, *and let* $\{\mathbf{A}_L\}$ *be the associated family of operators on* $[0, L]$ *with* β *homogeneous boundary conditions at* $x = 0$ *and* $\gamma = d - \beta$ *homogeneous boundary conditions at* $x = L$. *Then for any* $\varepsilon > 0$,

$$\lim_{L\to\infty} \sigma_\varepsilon(\mathbf{A}_L) = \sigma_\varepsilon(\mathbf{A}), \tag{10.14}$$

and thus

$$\lim_{\varepsilon\to 0}\lim_{L\to\infty} \sigma_\varepsilon(\mathbf{A}_L) = \sigma(\mathbf{A}). \tag{10.15}$$

Příklad:

$$\mathbf{A}u = u' + u'', \qquad f(k) = -\mathrm{i}k - k^2. \tag{10.11}$$

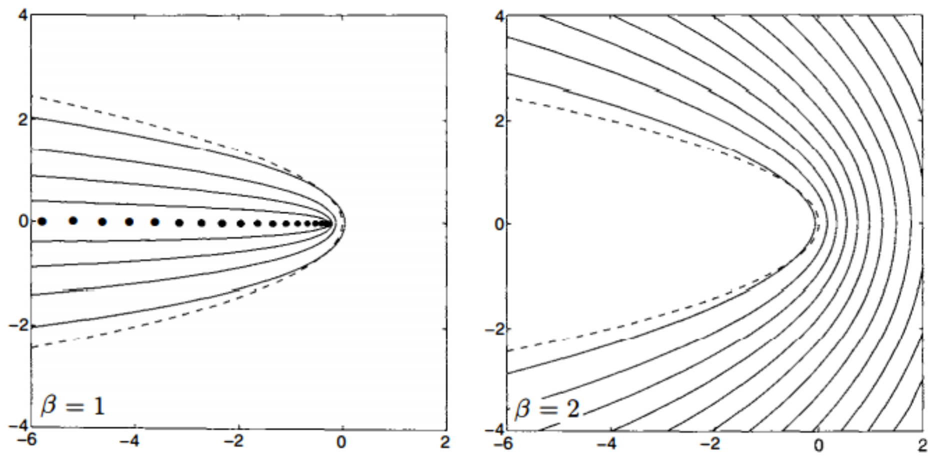


Figure 10.7: Spectrum and ε-pseudospectra of the advection-diffusion operator (10.11) on $[0, 24]$ for $\varepsilon = 10^{-1}, 10^{-2}, \ldots$ with one (left) and two (right) boundary conditions at $x = 0$.

Příklad:

$$\mathbf{A}u = -4u' + 6u'' - 15u''' - 12u^{(5)} - 2u^{(6)} \tag{10.12}$$

with symbol

$$f(k) = 4\mathrm{i}k - 6k^2 - 15\mathrm{i}k^3 + 12\mathrm{i}k^5 + 2k^6, \tag{10.13}$$

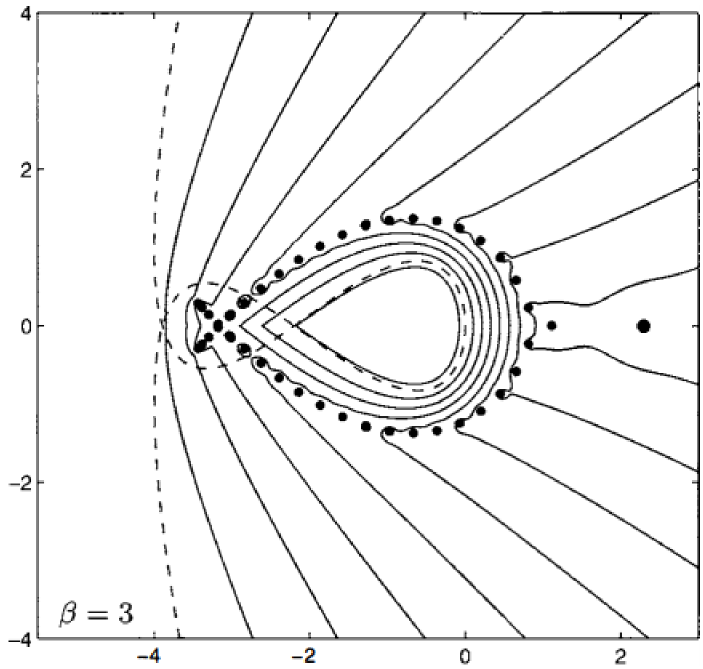


Figure 10.8: Spectrum and ε-pseudospectra of the sixth-order differential operator (10.12) on $[0, 120]$ with $\beta = \gamma = 3$ homogeneous boundary conditions at each endpoint, for $\varepsilon = 10^{-1}, 10^{-2}, \ldots, 10^{-8}$.