$$\sum_{n=1}^{+\infty} \underbrace{\frac{n x}{(1+x)(1+2x)\cdots(1+nx)}}_{u_n(x)} = (\check{R}) \text{ na } (0,+\infty)$$

$$0 < \frac{nx}{1+nx} < 1$$

$n>2$

$$\frac{1}{(1+x)(\ldots)\ldots(1+(n-2)x)(1+(n-1)x)} \leq \frac{1}{(n-2)(n-1)x^2}$$

$$|u_n(x)| \leq \frac{1}{(n-2)(n-1)x^2} \leq \frac{1}{(n-2)(n-1)} \frac{1}{\delta^2}$$

$(x \geq \delta > 0)$

a $\sum_{n=3}^{+\infty} \frac{1}{(n-2)(n-1)} \frac{1}{\delta^2} < +\infty \Rightarrow$ řada $(\check{R}) \rightrightarrows$ na $(\delta,+\infty)$ pro lib. $\delta > 0$.

BC - podm. pro řady: Nechť $u_n: J \to \mathbb{R}$. Pak

$$\sum_{n=1}^{+\infty} u_n \rightrightarrows \text{ na } J \Leftrightarrow \forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall k > n_0, p \in \mathbb{N} \; \forall x \in J: \left|\sum_{n=k+1}^{k+p} u_n(x)\right| < \varepsilon$$

$$\left|\sum_{n=1}^{k+p} u_n(x) - \sum_{n=1}^{k} u_n(x)\right|$$

$$\neg BC: \exists \varepsilon > 0, \forall n_0 \in \mathbb{N}, \exists k > n_0, p \in \mathbb{N}, x \in J: \left|\sum_{n=k+1}^{k+p} u_n(x)\right| \geq \varepsilon$$

$$\sum_{n=1}^{+\infty} \underbrace{\frac{nx}{(1+x)\quad(1+nx)}}_{u_n(x)} \qquad u_n\left(\tfrac{1}{n}\right) = \frac{1}{(1+\frac{1}{n})(1+\frac{2}{n})\quad \cdot 2} \geq \frac{1}{2^n}$$

$$u_n\left(\tfrac{1}{n^2}\right) = \frac{\frac{1}{n}}{(1+\frac{1}{n^2})(1+\frac{2}{n^2})(\cdot)\quad(1+\frac{1}{n})}$$

$$\geq \frac{\frac{1}{n}}{\underbrace{(1+\frac{1}{n})^n}_{\to e}} \xrightarrow{n\to+\infty} 0$$

$$u_j\left(\frac{1}{n^2}\right) = \frac{j\,\frac{1}{n^2}}{\left(1+\frac{1}{n^2}\right)\cdots\left(1+j\,\frac{1}{n^2}\right)} \geq \frac{\frac{1}{n}}{\left(1+j\,\frac{1}{n^2}\right)^n}$$

$j \in [n, 2n]$

$$\geq \frac{1}{n}\,\frac{1}{\underbrace{\left(1+\frac{2}{n}\right)^n}_{\xrightarrow[n\to+\infty]{} e^2}}$$

$\Rightarrow$ Pro dost velká n ($\exists n_0 \in \mathbb{N}, \forall n > n_0$):

$$\forall j \in \mathbb{N} \cap [n, 2n] : u_j\left(\frac{1}{n^2}\right) \geq \frac{1}{n}\,\frac{1}{2e^2}$$

$$\sum_{j=n+1}^{2n} u_j\left(\frac{1}{n^2}\right) \geq \frac{1}{n}\,\frac{1}{2e^2}\cdot n = \frac{1}{2e^2}$$

$\Rightarrow$ řada $\sum_{n=1}^{+\infty} u_n(x)$ nekonverguje st. na žádném okolí 0 $(0, \delta)$, $\delta > 0$ lib.

$$\sum_{n=1}^{+\infty} \frac{n}{n^2+1} x^n = (\tilde{R})$$

- $\forall x \in (-1,1)$ konverguje absolutně
- $x = 1$: diverguje
- $x = -1$: konverguje ale ne absolutně $\sum_{n=1}^{+\infty} \underbrace{\frac{n}{n^2+1}}_{\to 0 \text{ monotónně}} \underbrace{(-1)^n}_{\text{omez. č. součty}}$
- $|x| > 1$: diverguje (není splněna nutná podm.)

$[-1,1)$ $\quad \left|\frac{n}{n^2+1} x^n\right| \underset{|x|<\delta<1}{\leq} \frac{n}{n^2+1}\delta^n, \quad \sum_{n=1}^{+\infty} \frac{n}{n^2+1}\delta^n < +\infty$

$\underset{\text{Weierstrass}}{\Rightarrow} \quad (\tilde{R}) \rightrightarrows$ na $(-\delta, \delta)$ pro lib. $\delta > 0$.

---

na $\begin{matrix}(-1,0) \\ [\end{matrix} \ni x$: $\left|\sum_{n=0}^{N} x^n\right| = \left|\frac{1-x^{N+1}}{1-x}\right| \leq \frac{2}{1} \leq 2$

$\frac{n}{n^2+1} \searrow 0 \quad \Big(g(x) = \frac{x}{x^2+1},$

$g'(x) = \frac{1}{x^2+1} - \frac{x \cdot 2x}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} \Big)$

$\underset{\text{Dirichlet}}{\Rightarrow} \quad (\tilde{R}) \rightrightarrows$ na $[-1,0)$

$(0,1)$ $\quad u_n(x) = \frac{n}{n^2+1} x^n$, $\quad$ $x = 1 - \frac{1}{n}$, $\left(1-\frac{1}{n}\right)^n \to \frac{1}{e}, n \to +\infty$

$u_k\left(1-\frac{1}{n}\right) = \frac{k}{k^2+1}\left(1-\frac{1}{n}\right)^k \geq \cancel{\frac{k}{k^2+1}} \frac{n}{4n^2+1} \underbrace{\left(1-\frac{1}{n}\right)^{2n}}_{\to \left(\frac{1}{e}\right)^2}$

$k \in \mathbb{N} \cap [n, 2n]$

$\sum_{k=n+1}^{2n} u_k\left(1-\frac{1}{n}\right) \geq \frac{n^2}{4n^2+1}\left(1-\frac{1}{n}\right)^{2n} \xrightarrow{n \to +\infty} \frac{1}{4}\,\frac{1}{e^2}$

$\Rightarrow (\tilde{R}) \not\rightrightarrows$ na $(0,1)$

$$\sum_{n=1}^{+\infty} \frac{n}{n^2+1} x^n \operatorname{arctg}(nx) = (\check{R}_1)$$

Abelovo kritérium: na $(0,1)$ i $(-1,0)$

$\operatorname{arctg}(nx)$ je monotónní posl. fcí

je omezená

> 0 na $(0,1)$

< 0 na $(-1,0)$

$\Rightarrow (\check{R}_1)$ konv. stl. na $[-1, \delta)$; pro lib. $\delta > 0$ a $\delta < 1$.