Algebra II

Exercises for week 10

Problem 1. Fill out tables of operations $+$ and $\cdot$ of the 8-element finite field.

Problem 2. Prove that the mapping $x \mapsto x^p$ is an automorphism of any finite field of characteristic p.

Problem 3. Find an element that generates the multiplicative group of the finite field $\mathbb{Z}_3[x]/(x^2+1)$. Such elements are called primitive elements.

Problem 4 (from Allenby's "Rings, fields and groups")**.** Show that polynomials $p = x^4 + x + 1$ an $q = x^4 + x^3 + x^2 + x + 1$ are both irreducible over $\mathbb{Z}_2$. From theory, we know that $\mathbb{Z}_2[x]/(p)$ and $\mathbb{Z}_2[x]/(q)$ are isomorphic fields. Show that the "obvious" mapping $f/(p) \mapsto f/(q)$ is *not* an isomorphism of fields.