Tutorial 8

Problem 1. Let $n \in \mathbb{N}$. Let P be the (minimization version of) the entropy maximization problem

$$\begin{aligned} \text{minimize } & \sum_{i=1}^{n} x_i \ln x_i \\ \text{subject to } & a_1 x_1 + \cdots + a_n x_n = b \\ & x_1 + x_2 + \cdots + x_n = 1. \end{aligned}$$

State the dual problem to P and think about how to solve it analytically.

Problem 2. How does the dual problem to the quadratic program

$$\begin{aligned} \text{minimize } & x_1^2 + x_2^2 + x_3^2 - x_1 - x_2 - 2x_3 \\ \text{subject to } & x_1 + x_2 + x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

look like?

Problem 3. Duality works for cone programming, too. Follow the recipe of Lagrangian $\to$ Lagrange dual function to state the dual to the SDP in the standard form with one equality condition:

$$\begin{aligned} \text{minimize } & \operatorname{Tr}(CX) \\ \text{subject to } & X \succeq 0 \\ & \operatorname{Tr}(AX) = b, \end{aligned}$$

where C, A, b are fixed.

Problem 4 (duality in game theory)**.** Consider the following game of rock-paper-partial scissors:

Player 1 can play rock, paper, or scissors, while player 2 can only play rock and scissors. Both players reveal their choices at the same time. Whoever wins gains 1 point, the loser loses 1 point (a tie is worth 0 points).

A *strategy* for player 1 is a probability distribution over the 3 choices of moves; for player 2 it is a probability distribution over the 2 possible moves. Let $L(p, q)$ be the expected value of points gained by player 1 given player 1 strategy p and player 2 strategy q. A strategy p is *worst case optimal* for player 1 if $\inf_q L(p, q)$ is maximal possible (similarly for player 2).

Find a worst case optimal strategy for both players for this game.