Name:

Convex optimization

Homework set 11

Date due: **January 8 2020, 17:21**

Explain your reasoning in all the problems.

Problem	Pts max	Pts
1	2	
2	2	
3	2	
4	2	
5	2	
Σ	10	

Problem 1. We would like to reformulate the following problem (P) as a maximum likelihood estimate: "Given $\mathbf{y} \in \mathbb{R}^m$ and an $m \times n$ matrix A, find $\mathbf{x} \in \mathbb{R}^n$ so that the number of indices i where $\mathbf{y}$ and $A\mathbf{x}$ differ is minimal."

Following the trick from class, we can identify the penalty function ϕ in (P) and come up with a candidate for a probability density $p(v) \sim \exp(-\phi(v))$, however there is a serious issue. What is wrong with $p(v)$ as a density of a probability distribution?

Problem 2 (The standard Bayes theorem example revisited). Let us assume that 1 in 10 000 people have disease X and that we have a test T for this disease that has a 1 % false positive and false negative rate (that is, if a person has X then T answers “yes” 99 % of the time and if a person does not have X then T answers “no” 99 % of the time; T always answers “yes” or “no”).

We grabbed a random person and tested them; T says “yes.”

a) What does maximum likelihood estimate say about whether the person has X?

b) If we take the rate of X in population as our a priori probability for the person having X or not, what does MAP say about the whether the person has X?

Problem 3. Find a system of 20 linear inequalities that describe a polyhedron that cannot be given as a convex hull of fewer than 1000 points (that is, the polyhedron in question has at least 1000 vertices).

Problem 4. In this problem our goal will be to design an experiment that consists of 1000 measurements to estimate as precisely as possible $\mathbf{x} = (x_1, x_2)$. We can perform two types of measurements: Measurement A will return $x_1 + x_2 + w$ and measurement B will return $2x_1 + x_2 + w$ where w is (in both cases) noise (independent identically distributed between measurements) with mean 0 and variance 1.

How many which measurements should we do to minimize the volume of the confidence ellipsoid? Solve the problem by hand, not by a computer program (it can be done).

Problem 5. We have a coin with an unknown probability x of landing heads. Our a priori probability distribution for x has density $p(x) = C\exp(-4(x - 1/2)^2)$ for $x \in (0,1)$ a 0 for other values of x (here $C > 0$ is an uninteresting normalization constant). We observed 12 coin tosses: 7 heads and 5 tails.

Formulate the problem of finding the best a posteriori estimate for x as a convex optimization problem and solve it using a computer. Explain here what problem are you solving and which x you got. You do not have to submit your code.

You can consult with your friends when solving the homework, but you have to **write** your solutions (including Python code) **on your own** and **do not show your fininished solutions** to your peers before the due date.