Name:

Convex optimization

Homework set 12

Date due: **January 8 2020, 17:21**

Explain your reasoning in all the problems.

Problem	Pts max	Pts
1	2	
2	2	
3	2	
4	4	
Σ	10	

Problem 1. Find two $n \in \mathbb{N}$ and two matrices $P, Q \in S^n$ such that $P \succeq Q$, but $\det P < \det Q$.

Note: This would be impossible for matrices from S_+^n.

Problem 2. Let us explore how the gradient descent behaves for the function $f(x_1, x_2) = 1/2(x_1^2 + \gamma x_2^2)$ with initial point $\mathbf{x}^{(0)} = (\gamma, 1)$, where $\gamma > 0$ is a parameter. Assume for simplicity that instead of BLS we use exact search to find t in our gradient descend; that is we always choose t so that $f(\mathbf{x} + t\boldsymbol{\Delta}\mathbf{x})$ is minimal.

Prove that in the k-th step of gradient descent we will have

$$\mathbf{x}^{(k)} = \left(\gamma \left(\frac{\gamma - 1}{\gamma + 1}\right)^k, \left(-\frac{\gamma - 1}{\gamma + 1}\right)^k\right).$$

What does this mean for the convergence rate when γ is large?

Problem 3 (Uniqueness of projection)**.** Let $\|\cdot\|$ be a norm in $\mathbb{R}^n$, let $C \subset \mathbb{R}^n$ be a nonempty closed convex set, and let $\mathbf{x}_0 \in \mathbb{R}^n$ be a point. Show that there exists *exactly one* $\mathbf{x} \in C$ that minimizes $\|\mathbf{x} - \mathbf{x}_0\|$.

Problem 4. Find for each $n \in \mathbb{N}$ a formula for the Löwner-John ellipsoid $\mathcal{E}$ of the n-dimensional simplex

$$C = \left\{ \sum_{i=1}^{n} t_i \mathbf{e}_i \colon \sum_{i=1}^{n} t_i \leq 1, \, \mathbf{t} \succeq \mathbf{0} \right\}.$$