Absorbing sets and where to find them

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What is absorption

Definition (Libor Barto, Marcin Kozik)

Let $B \leq A$ be algebras. We say that B absorbs A if there exists a term t in A such that for any $b_1, \dots, b_n \in B, a \in A$ we have:

$$
\begin{aligned}
t(a, a, a, \dots, a) &= a \\
t(a, b_2, b_3, \dots, b_n) &\in B \\
t(b_1, a, b_3, \dots, b_n) &\in B \\
&\vdots \\
t(b_1, b_2, \dots, b_{n-1}, a) &\in B
\end{aligned}
$$

Ok, but what *is* absorption?

- If 0 is the minimal element of a finite semilattice $(L, \wedge)$ then $\{0\}$ absorbs L; absorption term is $t(x_1, x_2) = x_1 \wedge x_2$.
- If A is an algebra with a majority term m then every singleton is an absorbing subalgebra; absorption term is m.
- If A is an algebra then always $A \trianglelefteq A$.
- If A is an abelian group then A has no proper absorbing subalgebra.

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How to use it

- Absorption is useful for induction-style arguments (see Libor Barto and Marcin Kozik's CSP results).
- If we can absorb, we can often make counterexamples smaller.
- Example: Let P be a connected poset, let $Q \trianglelefteq P$. Then Q is also a connected poset.

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Where to find it

- People have been using absorption under different names:
- B. Larose, C. Loten, C. Tardif: A finite relational structure $\mathbb{A}$ has first order definable CSP iff $R \trianglelefteq A^k$ for every k-ary R relation of $\mathbb{A}$.
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Thanks for your attention.