1st problem set

Problém 1. Let $\mathbf{A}$ be a semilattice (ie. algebra with one commutative associative idempotent binary operation) on at least 2 elements. Prove that $\mathbf{A}$ is not Abelian.

Problém 2. Prove that a ring $\mathbf{R}$ is Abelian if and only if for all $r, s \in R$ we have $rs = 0$. We don't assume that rings have the element 1.

Problém 3. Prove that a group $\mathbf{G}$ is Abelian if and only if it is commutative.

Problém 4. Consider the following lazy term condition (which is not actually used outside of this exercise): For every t *basic operation* of $\mathbf{A}$ (i.e. operation from the signature of $\mathbf{A}$), every $x, y \in A$ and every $\mathbf{u}, \mathbf{w} \in A^{n-1}$ (where n is the arity of t) the implications

$$t(x, u_1, \dots, u_n) = t(x, v_1, \dots, v_n) \Rightarrow t(y, u_1, \dots, u_n) = t(y, v_1, \dots, v_n)$$
$$t(u_1, x, u_2, \dots, u_n) = t(v_1, x, v_2, \dots, v_n) \Rightarrow t(u_1, y, u_2, \dots, u_n) = t(v_1, y, v_2, \dots, v_n)$$
$$\vdots$$
$$t(u_1, \dots, u_n, x) = t(v_1, \dots, v_n, x) \Rightarrow t(u_1, \dots, u_n, y) = t(v_1, \dots, v_n, y)$$

hold.

Observe that all Abelian algebras satisfy the lazy term condition. Why did we define the lazy term condition with n implications instead of one implication for the term condition? Prove that all groups satisfy the lazy term condition.

Problém 5. Let $\mathbf{A}$ be an algebra with a Maltsev polynomial m that is central in all polynomials of $\mathbf{A}$. Pick $0 \in A$ and define operations $x + y = m(x, 0, y)$, $-x = m(0, x, 0)$. Prove that $(A, +, -, 0)$ is a commutative group.