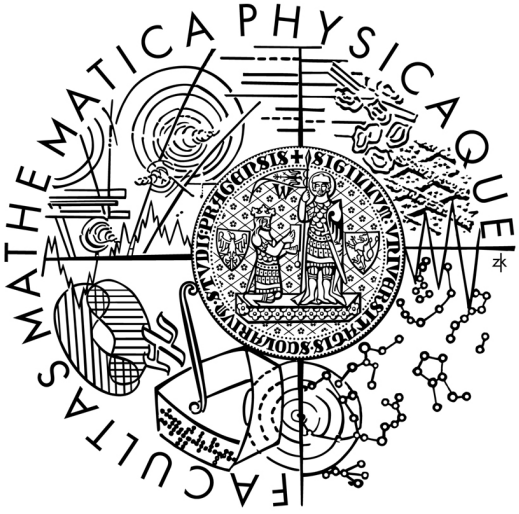
Charles University in Prague
Faculty of Mathematics and Physics

23rd Annual Student Conference

Week of Doctoral Students 2014



Book of Abstracts

of the

Week of Doctoral Students of the School of Mathematics, 2014 June 5-6, 2014

Sokolovská 83
186 75 Praha 8

Editace v LaTeXu a grafická úprava: Andrej Živčák, Petr Pošta, Mirko Rokyta

http://www.karlin.mff.cuni.cz/~rokyta/WDS2014/
http://www.mff.cuni.cz/veda/konference/wds/

Vytisklo Reprostředisko MFF UK Praha

Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

During the February meeting of the authorities of the Doctoral Study Committees (RDSO) with the Vice-Dean for Mathematics it was decided that the WDS of the School of Mathematics (`http://www.karlin.mff.cuni.cz/~rokyta/WDS2014/`) will be organized during the first week of June 2014 as a two-day conference, in the framework, and as a continuation of, the (23th) WDS of the Faculty of Mathematics and Physics (`http://www.mff.cuni.cz/veda/konference/wds/`).

We are pleased that as many as 28 students have registered for the conference. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

Prague, June 2, 2014

doc. RNDr. Mirko Rokyta, CSc.
Vice-Dean for Mathematics
Faculty of Mathematics and Physics
Charles University Prague

Contents

4M8 – Obecné otázky matematiky a informatiky

4F11 – Matematické a počítačové modelování

Products of self-small Abelian groups

Mgr. Josef Dvořák

Obor studia: 4M1 – Algebra, teorie čísel a matematická logika
Ročník: 1.
Školitel: Mgr. Jan Žemlička, Ph.D.

Abstract

An Abelian group A is called self-small if direct sums of copies of A commute with the covariant $\mathrm{Hom}(A,-)$ functor. An elementary example of a non-self-small countable product of self-small Abelian groups without non-zero homomorphisms between different ones will be presented and a criterion of self-smallness of a finite product of modules will be given. Furthermore, an overview of the notion of quotient category, which serves as the modern language for theory of self-small groups, will be presented.

Products of small modules

Mgr. Peter Kálnai

Obor studia: 4M1 – Algebra, teorie čísel a matematická logika
Ročník: 1.
Školitel: Mgr. Jan Žemlička, Ph.D.

Abstract

A module is said to be small if it is not a union of strictly increasing infinite countable chain of submodules. We show that the class of all small modules over self-injective purely infinite ring is closed under direct products whenever there exists no strongly inaccessible cardinal.

Index theorem for elliptic operators

Mgr. Marek Bernát

Obor studia: 4M2 – Geometrie a topologie, globální analýza a obecné struktury
Ročník: 1.
Školitel: RNDr. Svatopluk Krýsl, Ph.D.

Abstract

We describe the theory of Atiyah and Singer that allows one to obtain certain analytic information on elliptic operators purely by topological means. The operators are assumed to act between sections of vector bundles and the topological data is either of K-theoretical or cohomological nature. We will provide some historical perspective and motivation leading to the theorem. Then we introduce all the relevant machinery needed to formulate the theorem, such as K-theory, Chern classes, Thom isomorphism and Todd class. Afterwards we will briefly sketch the original proof of the theorem and mention some of its main corollaries and applications.

Heisenberg group and Sobolev-like function spaces

Mgr. Martin Franců

Obor studia: 4M3 – Matematická analýza
Ročník: 2.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

Heisenberg group can be realised as $\mathbb{R}^3$ with proper operation. This defines a tangent bundle with respect to this operation. By picking certain (horizontal) subbundle of tangent bundle and consider admissible only paths with tangent vector in this subbundle, we obtain the so-called horizontal geometry. This geometry leads to rich structure (metrics, topology, etc.) and also to function spaces analogous to Sobolev spaces of first order. Our interest will be in examining possibilities in defining Sobolev-like function spaces in the setting of r.i. spaces.

Convolution in weighted Lorentz spaces

Mgr. Martin Křepela

Obor studia: 4M3 – Matematická analýza
Ročník: 3.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

Young-type convolution inequalities of the form

$$\|f * g\|_{\Gamma^q(w)} \leq C\|f\|_X\|g\|_Y$$

are proved, where X is one of the weighted Lorentz-type spaces $\Lambda^p(v)$, $S^p(v)$ or $\Gamma^p(v)$. The norm of the optimal rearrangement-invariant space Y satisfying this inequality is characterized.

Face of (I)-envelope of the ball in a concrete class of Banach spaces

Mgr. Jindřich Lechner

Obor studia: 4M3 – Matematická analýza
Ročník: 1.
Školitel: doc. RNDr. Ondřej Kalenda, Ph.D., DSc.

Abstract

I will talk about the following theorem: If K is a totally disconnected compact space such that the algebra of its clopen subsets has the Subsequential Completeness Property, then $C(K)$ is 1-Grothendieck. A key notion in the proof is so called (I)-envelope. This term was introduced by Kalenda and is rooted in the notion of (I)-generating established by Fonf and Lindenstrauss. The proof is almost entirely based on ideas contained in the Kalenda theorem which says that (I)-envelope of B_{ℓ_∞} is the whole bidual ball. And the conclusion one gets by using the Bendová characterization the quantitative Grothendieck property in the language of (I)-envelopes.

A compressible Navier-Stokes system with entropy transport

Mgr. Martin Michálek

Obor studia: 4M3 – Matematická analýza
Ročník: 1.
Školitel: prof. RNDr. Eduard Feireisl, DrSc.

Abstract

In simplified thermodynamical models for ideal gas are compressible Navier-Stokes equations (for unknowns ρ and $\vec{u}$) supplemented with transport equation for entropy s. The pressure term in the momentum equation has form $p(\rho,s) = e^s \rho^\gamma$. A proof of existence of a global weak solution for this system was sketched by P.-L. Lions under a non-physical hypothesis on γ, namely $\gamma > 9/5$. We show some unpleasant mathematical features when we want to deal with smaller coefficients γ. Existence for physically reasonable $\gamma = 5/3$ is required.

Lineability of nowhere monotone measures

Mgr. Petr Petráček

Obor studia: 4M3 – Matematická analýza
Ročník: 3.
Školitel: prof. RNDr. Jaroslav Lukeš, DrSc.

Abstract

We say that a finite signed Radon measure on the real space $\mathbb{R}^d$ is *nowhere monotone* if

$$\mu^+(G) > 0,$$
$$\mu^-(G) > 0,$$

holds for every non-empty open set $G \subset \mathbb{R}^d$. We will show that there exists a dense vector subspace of the space of finite signed Radon measures on $\mathbb{R}^d$ that are almost everywhere differentiable with respect to the d-dimensional Lebesgue measure such that every non-zero element of this subspace is a nowhere monotone measure (in literature, this is referred to as *dense maximal lineability*). Unlike many similar results a major part of the proof is non-constructive.

This result is a recent addition to the increasingly popular research on finding large vector structures in generally non-linear settings.

Nonstandard analysis of global attractors

Mgr. Jakub Slavík

Obor studia: 4M3 – Matematická analýza
Ročník: 1.
Školitel: doc. RNDr. Dalibor Pražák, Ph.D.

Abstract

We formulate the key concepts of the theory of abstract dynamical systems in the language of nonstandard analysis. We try to illustrate that certain attributes like compactness of the ω-limit set can be achieved in the nonstandard setting with almost no effort. In particular we give an alternative proof to the characterization of global attractors due to J. M. Ball, find a nonstandard sufficient condition for connectedness using internal discrete infinitesimal paths and use it to study the connectedness of ω-limit sets. The key observation is that the ω-limit set can be written as a standard part of a suitable internal set.

Lebesgue differentiation theorem in rearrangement-invariant spaces

Mgr. Lenka Slavíková

Obor studia: 4M3 – Matematická analýza
Ročník: 2.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

The classical Lebesgue differentiation theorem tells us that whenever u is a locally integrable function on an open set $\Omega \subset \mathbb{R}^n$, then for almost every $x \in \Omega$

$$\lim_{r \to 0_+} \frac{1}{|B(x,r)|} \int_{B(x,r)} |u(y) - u(x)|\, dy = 0. \tag{1}$$

We study a variant of the theorem when the integral average in (1) is replaced by the average with respect to a rearrangement-invariant norm. We characterize those rearrangement-invariant norms for which this generalization of the Lebesgue differentiation theorem holds, and we also provide specific examples of such norms.

Normability of weighted Lorentz spaces

Mgr. Filip Soudský

Obor studia: 4M3 – Matematická analýza
Ročník: 3.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

We shall study normability of Lorentz spaces of the type Λ and Γ. We shall first summarize temporary knowledge of this topic and then add a new result.

Volterra processes

Mgr. Petr Čoupek

Obor studia: 4M4 – Pravděpodobnost a matematická statistika
Ročník: 1.
Školitel: prof. RNDr. Bohdan Maslowski, DrSc.

Abstract

Volterra processes are Gaussian processes for which the strong representation $B_t = \int_0^t K(t,r)\mathrm{d}W_r$ with $\mathscr{F}_t^B = \mathscr{F}_t^W$ holds. We consider strictly regular Volterra processes with vanishing kernel K and define a stochastic integral which takes B_t as a driving process. The main result is that the space of B-integrable deterministic functions can be embedded into a suitable Lebesgue space. This is further used to show the measurability of a mild solution of a given stochastic evolution equation driven by a cylindrical analogue of the Volterra process.

Testing of random-field model for random marked closed sets

Mgr. Antonín Koubek

Obor studia: 4M4 – Pravděpodobnost a matematická statistika
Ročník: 1.
Školitel: doc. RNDr. Zbyněk Pawlas, Ph.D.

Abstract

We define a random marked closed set, which is a stochastic process defined on a random closed set in $\mathbb{R}^d$. It is a generalization of both random fields and marked point processes. As a motivating example, an application in meteorology is presented. Our main theoretical aim is to test whether the marks are independent of the set (so called random-field model). For this purpose we can estimate some second-order characteristics and apply Monte Carlo test. Another possibility is to generalize tests that are already developed for marked point processes.

Local ensemble transform Kalman filter

Mgr. Marie Turčičová

Obor studia: 4M4 – Pravděpodobnost a matematická statistika
Ročník: 1.
Školitel: RNDr. Kryštof Eben, CSc.

Abstract

Data assimilation is an iterative method of estimating the state of a dynamical system with the help of observations. Typically we have a prior estimate of the state resulting from evolution of the system in time; this prior estimate is "updated" so as to be closer to current observations. Often we also want to control the size of departures from the prior estimate, in order to keep physical consistence of the state. A large group of assimilation methods is based on Kalman filter and its generalizations, other methods use a variational approach.

Data assimilation techniques are widely used in environmental modelling, in particular in numerical weather prediction. Here we face some challenges arising from the huge dimension of the state vector and from nonlinearity of the dynamics.

One of the Kalman filter adjustments that addresses these two problems is called Ensemble Kalman Filter (EnKF). In EnKF the current state of the system and its uncertainty are estimated via their sample counterparts calculated from a set of forecasts (which may be viewed as candidate scenarios) called ensemble. The accuracy and computational efficiency of ensemble methods can be further improved via localization, where the update step of the EnKF is done locally in space. We will focus on one particular version of the local filter, called Local Ensemble Transform Kalman Filter. In this method each local region is processed independently using only observations within that region. This permits efficient parallel implementation of the filter and suppresses spurious long-range correlations which arise as a result of a limited ensemble size.

Stability analysis of the space-time DGM

Mgr. Monika Balázsová

Obor studia: 4M6 – Vědecko-technické výpočty
Ročník: 1.
Školitel: prof. RNDr. Miloslav Feistauer, DrSc., dr. h. c.

Abstract

The subject of the presentation is the analysis of the space-time discontinuous Galerkin method for the solution of nonstationary, nonlinear, convection-diffusion problems. In the formulation of the numerical scheme, the nonsymmetric, symmetric and incomplete versions of the discretization of diffusion terms and interior and boundary penalty are used. Then error estimates are briefly characterized. The main attention is paid to the investigation of unconditional stability of the method. An important tool is the concept of the discrete characteristic function. Theoretical results are accompanied by numerical experiments.

On the properties of Krylov subspaces in finite precision CG computations

Mgr. Tomáš Gergelits

Obor studia: 4M6 – Vědecko-technické výpočty
Ročník: 1.
Školitel: prof. Ing. Zdeněk Strakoš, DrSc.

Abstract

The method of conjugate gradients (CG) for solving linear systems of algebraic equations

$$Ax = b, \qquad A \in \mathbb{F}^{N\times N},\ b \in \mathbb{F}^{N}, \quad \text{where } \mathbb{F} \text{ is } \mathbb{C} \text{ or } \mathbb{R},$$

with a large, sparse and Hermitian and positive definite matrix A is computationally based on short recurrences. Assuming exact arithmetic, they ensure the global orthogonality of the residual vectors, which span at the l-th step the l-dimensional Krylov subspace

$$\mathscr{K}_l(A, r_0) = \text{span}\left\{r_0, Ar_0, A^2r_0, \ldots, A^{l-1}r_0\right\}.$$

In practical computations, however, the use of short recurrences inevitably leads to the loss of the global orthogonality and even linear independence among the computed residual vectors. Consequently, the computed Krylov subspaces can be "rank-deficient" which causes in finite precision CG computations a significant delay of convergence. In this contribution we address the question how the Krylov subspaces generated by the CG method in finite precision arithmetic differ from their exact arithmetic counterparts. After recalling the results published previously we concentrate on the situation with a significant delay of convergence and describe the distance between Krylov subspaces of the appropriately determined equal dimension. We observe that the finite precision Krylov subspaces in this comparison do not substantially deviate from the exact Krylov subspaces.

Regularization via noise revealing in Golub-Kahan bidiagonalization

Mgr. Marie Kubínová

Obor studia: 4M6 – Vědecko-technické výpočty
Ročník: 1.
Školitel: prof. Ing. Zdeněk Strakoš, DrSc.

Abstract

We consider a linear inverse problem $Ax \approx b$, where A is a linear operator with smoothing property and b represents an observation vector polluted by noise. Due to the smoothing property of A, these problems are typically *ill-posed*, meaning that noise in the data, especially its high-frequency components, may give rise to significant errors in computed approximations of x. It was shown in [Hnětynková, Plešinger, Strakoš, 2009] that high-frequency noise reveals during the Golub-Kahan iterative bidiagonalization in the left bidiagonalization vectors. In the talk we will present a way of determining the iteration with optimal noise revealing property and suggest reduction of the high-frequency part of the noise by subtracting the corresponding (properly scaled) left bidiagonalization vector from the data vector b. We will demonstrate the regularizing properties of the proposed noise reduction and explain its relation to projection methods.

Integral model for viscoelastic fluids

Mgr. Ivan Soukup

Obor studia: 4M6 – Vědecko-technické výpočty
Ročník: 2.
Školitel: prof. RNDr. Vít Dolejší, Ph.D., DSc.

Abstract

We briefly introduce an integral nonlinear model describing flow of a viscoelastic fluid and show our results covering existence, uniqueness and regularity of its solutions. We also point out the main steps of corresponding proofs.

Numerical solution of algebraic systems arising from the discontinuous Galerkin discretization of PDEs by p-multigrid

Mgr. Andrej Živčák

Obor studia: 4M6 – Vědecko-technické výpočty
Ročník: 2.
Školitel: prof. RNDr. Vít Dolejší, Ph.D., DSc.

Abstract

We deal with the discontinuous Galerkin (DG) method applied to the numerical solution of partial differential equations, namely to CFD.

The DG method is based on piecewise polynomial but discontinuous approximation, which allows us to simply construct hierarchical basis functions locally for each element. The DG discretization leads to the necessity to solve large (non-) linear algebraic systems. Among the most efficient techniques for the numerical solution of algebraic systems belong the so-called *multigrid methods*.

Multigrid methods are based on coarser representations of the discretized problem. It can be used for solving linear as well as nonlinear problems. Very well known and widely used h-multigrid is based on geometrical hierarchy of computational meshes. However, for the DG discretization, more suitable approach is the so-called p-multigrid, where a hierarchy of discretization spaces with respect to polynomial approximation degree p is considered.

Projection operators, which carry out the restriction and prolongation, depend on choice of basis functions. Due to the locality of a basis function in the DG method we get local projection operators. Their form is very simple in the case of orthonormal basis functions and therefore an effortless implementation can be used.

We describe the application of the p-multigrid to the numerical solution of linear algebraic systems arising from the DG discretization. We present the restriction and prolongation operators and discuss several solution strategies.

Numerical solution of a new hydrodynamic model of flocking

Mgr. Andrea Živčáková

Obor studia: 4M6 – Vědecko-technické výpočty
Ročník: 1.
Školitel: RNDr. Václav Kučera, Ph.D.

Abstract

Our work deals with the numerical solution of a new hydrodynamic model describing the flocking dynamics. The flocking dynamics is the phenomenon, when self-propelled individuals organize into an ordered motion using only limited environmental information and simple rules, e.g. flocking of birds, schooling of fishes, swarming of bacteria, etc.

The studied model of Fornasier et al. (2010) is a hydrodynamic limit of the well known Cucker-Smale model - the compressible Euler equations with an additional nonlocal nonlinear source term. Because of complexity of the model, we focus only on the one-dimensional case. For the numerical solution we use a suitably modified semi-implicit discontinuous Galerkin method. Our work represents the first thorough attempt to solve these equations numerically.

Software for computer aided teaching of geometry

Mgr. Eliška Bartošová

Obor studia: 4M8 – Obecné otázky matematiky a informatiky
Ročník: 1.
Školitel: RNDr. Pavel Surynek, Ph.D.

Abstract

The presentation addresses utilization of computers for teaching of geometry and for improving spatial imagination. I will summarize available data on usage of computers and software equipment for teaching of geometry. These data will be supplemented by results of my own study carried out on several high schools. Then an overview of existing programs usable for teaching of geometry will be provided. At last, I will introduce a complex system for teaching and exercising geometry which I'm working on – the Lisa System. It consists of a computer program – editor specialized for descriptive geometry, and a set of exercises for this program.

April Fools' Day science pranks

Mgr. Tereza Bártlová

Obor studia: 4M8 – Obecné otázky matematiky a informatiky
Ročník: 2.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

It seems that you can't believe anything you read or hear on April Fools' Day anymore, as many respectable media outlets and organizations try to outdo themselves with an outlandish news story. Scientists and scientific journals are at an advantage when it comes to fooling the rest of the world on April 1st. For one thing, the average reader is optimistic, and therefore liable to believe anything amazing when it comes to scientific discoveries. For another, no one expects a stereotypically dry, robotic scientist to pull a prank. If anything, the faster news cycle and the instantaneous nature of the Internet have fueled even more ridiculous April Fools' pranks, which can now travel around the globe in a flash. We will focus just on a few classic jokes in the science world from April Fools' Day's past.

Origins of geodesy and surveying

Mgr. Petra Plichtová

Obor studia: 4M8 – Obecné otázky matematiky a informatiky
Ročník: 1.
Školitel: doc. Ing. Pavel Hánek, CSc.

Abstract

We introduce geodesy as a scientific field, the surveyors's work and what the surveying and the geometry have in common. We'll talk about the origins of geodesy, focusing on ancient Greeks and their perception about Earth. We will then discuss attempts to measure the size of the Earth and ways to determine the latitude and longitude. Although these results are not precise from today's perspective, they still provide interesting insight into the history of geodesy. We will conclude with overview of basic geometry properties used in geodesy.

High-school combinatorics based on problem-solving

Mgr. Pavel Šalom

Obor studia: 4M8 – Obecné otázky matematiky a informatiky
Ročník: 2.
Školitel: doc. RNDr. Oldřich Odvárko, DrSc.

Abstract

This contribution deals with the conception of combinatorics for high schools. The aim of this conception is to avoid acquiring a purely formal knowledge. We have been inspired by the conception of professor Hejný's textbooks for elementary schools. We have designed four environments in which we want to build up the combinatorics. These environments are Airlines, Tournaments, Translations and Games. Sample problems from each environment are a part of the contribution.

A posteriori error estimates for a class of nonlinear PDE problems involving steady flows of implicitly constituted fluids

Mgr. Jan Blechta

Obor studia: 4F11 – Matematické a počítačové modelování
Ročník: 1.
Školitel: prof. RNDr. Josef Málek, CSc., DSc.

Abstract

We present a review of primal-based a posteriori error estimates. The framework provides a way to bound some appropriate error measure of approximate solution for quite a large class of PDEs without actually knowing the exact solution. Most recent results by M. Vohralík and A. Ern provide, for particular PDEs, desired properties of the estimates, like local efficiency of a bound (local equivalence of the bound with the actual error up to some constructible constant), asymptotic exactness (convergence of the bound to actual error) and robustness w.r.t. polynomial degree of the approximation or other parameters. The apparatus requires quite a weak conditions on the approximation – in particular it does not need to be constructed using conforming FE method; other non-conforming methods, like DGM, FVM, may work. Then the estimation procedure requires solving just local problems which makes the estimation cheap. One can observe where the distinct components (discretization, linearization, algebraic) of the estimator come from to adaptively adjust approximate calculation.

Computer modelling in plasma physics

Mgr. Jakub Hromádka

Obor studia: 4F11 – Matematické a počítačové modelování
Ročník: 1.
Školitel: prof. RNDr. Rudolf Hrach, DrSc.

Abstract

Computer modelling became powerful tool in contemporary research in plasma physics. Plasma can be described via different kinds of mathematical models. Some phenomena in plasma can be properly treated only by kinetic theory where distribution functions of all particle species are searched for. This approach often leads to complicated integro-differential equations. Instead, molecular dynamics method is used to solve equations of motion of large number of charged particles that plasma consists of. Unfortunately, such an approach is computationally demanding so the most of problems is simplified and solved in 2D geometry. Since the motion of charged particles in electromagnetic fields is generally three dimensional, it is highly desirable to implement fully three dimensional computer models that could bring valuable results for better understanding of processes in plasma probe diagnostics, especially in high-temperature plasma.

Our contribution summarizes modelling techniques used in plasma physics, presents some results obtained by 2D particle model in multicomponent plasma and tries to explain the concept of 3D hybrid models that seem to be the right way to reach precise results in reasonable computation time.

Modelling and analysis of reactive systems

Mgr. Vít Orava

Obor studia: 4F11 – Matematické a počítačové modelování
Ročník: 1.
Školitel: RNDr. Jaroslav Hron, Ph.D.

Abstract

I will present thermodynamically consistent modelling of reactive convection-diffusion mixture undergoing heterogeneous catalysis on a catalytic boundaries. Two posible description – Fick's and Maxwell-Stefan's – of diffusive fluxes in setting of Class I mixture model will be discussed. These kinds of the models are described by nonlinear system of convection-diffusion PDEs in bulk and reaction-diffusion PDEs on the catalytic surfaces coupled by nonlinear BC in the first case and nonlinear source term in the second case. Finally, some qualitative behaviour of the weak solution of the system such as existence and uniqueness will be mentioned.

Imaging, modeling and computations of hemodynamics in cerebral aneurysms

Mgr. Helena Švihlová

Obor studia: 4F11 – Matematické a počítačové modelování
Ročník: 1.
Školitel: RNDr. Ing. Jaroslav Hron, Ph.D.

Abstract

With increasing popularity of imaging techniques such as computed tomography an aneurysm is often discovered on the brain artery. The aneurysm is a local extension of a vessel and it is dangerous only in the case of rupture. The need of accurate computation of the velocity and pressure fields in patient specific geometries is motivated exactly by the question which aneurysm has tendency to rupture.

Even if we consider incompressible Newtonian fluid, the problem is quite challenging. We will start with the question of correct boundary conditions on the vessel walls, which can have significant influence on the usual measures. We will then discuss the appropriate hemodynamic parameters which affect the growth or rupture of the aneurysm. Finally, we will mention the possibility of involvement of interaction with elastic wall to the model.